THE USE OF ICT IN PRIMARY MATHEMATICS IN CYPRUS: THE CASE OF GEOGEBRA

A THESIS SUBMITTED IN SATISFACTION OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF PHILOSOPHY IN EDUCATION (INTERNATIONAL PERSPECTIVES IN MATHEMATICS EDUCATION)

NAME: IRINI CHRYSANTHOU

SUPERVISOR: DR TIM ROWLAND

SUBMISSION DATE: 14TH JULY 2008
Declaration

In accordance with Regulation 8 of the General Regulations for the MPhil Degree (one-year course) I declare that this thesis entitled ‘THE USE OF ICT IN PRIMARY MATHEMATICS IN CYPRUS: THE CASE OF GEOGEBRA’ is substantially my own work. Where reference is made to the works of others the extent to which that work has been used is indicated and duly acknowledged in the text and bibliography.

The length of my thesis, excluding Declaration, Acknowledgements, List of tables and figures, Table of Contents, Foot-notes, References and Appendices, is 20286 words.
Acknowledgements

First of all, I would like to express my sincere thanks to my supervisor Dr Tim Rowland for his intuitive assistance, encouragement and guidance throughout this year of study in Cambridge. With his expertise and experience he gave me comments that always helped me to move forward with my work. I am also very grateful to the teacher who participated in the research for her invaluable contribution. Despite the high intrusion degree of the research she offered her best efforts and cooperation in completing the research. I would also like to thank the school and in particular the 16 pupils that participated in the study. Finally I want to thank my family and friends for their support and encouragement throughout this challenging process.
Abstract

As I primary school teacher, I was interested in conducting an empirical study using GeoGebra - a free open-source dynamic software for mathematics teaching and learning that links geometry and algebra into a single easy-to-use package - exploring the potentials of this software for teaching primary mathematics and the implications of its use for classroom practice. Within this framework I set out to explore how the classroom dynamics formed, the potential of Geogebra for mathematics teaching and learning, students’ learning attitudes and the feasibility of its use in the primary classroom. The research is based on a social constructivist view of learning and the methodology used is a case study. The research was conducted in Cyprus and involved a teacher with her class of 16 students. In this research I used several data collection techniques to ensure triangulation of the data and to explore different perspectives. My data consisted of classroom observations, teacher interviews and student questionnaires. From the data it seems that the introduction of Geogebra influences the educational practice in three dimensions, namely: classroom practice, cognitive development and learning attitudes.
List of tables and figures

Figure 2.1: Screenshot from a Geogebra window…………………………………………..20

Figure 2.2: List of all Geogebra tools (Prepared by Mark Dawes for the Geogebra workshop on 23.06.2008 at Comberton Village College)…………………………………21

Figure 3.1: Example of an application used in the first lesson………………………………..28

Figure 3.2: Example of an application used in the second lesson………………………………29

Figure 3.3: Example of an application used in the third lesson………………………………29
# Table of contents

Declaration................................................................................................................................. 2
Acknowledgements...................................................................................................................... 3
Abstract..................................................................................................................................... 4
List of tables and figures.............................................................................................................. 5
Table of contents.......................................................................................................................... 6
Chapter 1: Introduction............................................................................................................... 9
Chapter 2: Literature review ..................................................................................................... 12
  Potentials of ICT for mathematics education........................................................................ 12
  Integrating technology into mathematics teaching............................................................... 15
    *Why integrate ICT in mathematics education?* ................................................................. 16
    *Teacher’s role*...................................................................................................................... 17
Means of ICT in the classroom .................................................................................................. 20
  *Using computers in classrooms* ......................................................................................... 20
  *Types of software*............................................................................................................... 23
The Dynamic Mathematics Software GeoGebra ..................................................................... 26
  *What is GeoGebra?* ............................................................................................................. 26
  *Why is GeoGebra different?* ............................................................................................. 26
  *What has GeoGebra to offer?* .......................................................................................... 27
  *Teaching Mathematics with GeoGebra* ............................................................................. 28
Research questions.................................................................................................................... 30
Chapter 3: Research Design..................................................................................................... 31
  Epistemology.......................................................................................................................... 31
Theoretical perspective .................................................................................................................. 32
Methodology .................................................................................................................................. 33
  Participants and procedure ........................................................................................................... 34
  Pilot study ..................................................................................................................................... 38
  Ethical issues ................................................................................................................................. 39
Methods ........................................................................................................................................ 39
  Observations ................................................................................................................................. 40
  Interviews ...................................................................................................................................... 41
  Questionnaires .............................................................................................................................. 43
Transcribing the data ..................................................................................................................... 44
Analyzing the data .......................................................................................................................... 45
Chapter 4: Findings ....................................................................................................................... 47
  Classroom organization and management ..................................................................................... 49
    Break from routine ...................................................................................................................... 49
    Facilitating classroom activity .................................................................................................... 49
    Enhancing productivity of classroom activity ............................................................................. 52
    Teacher as resource .................................................................................................................... 53
Cognitive amplification ................................................................................................................... 56
  Extenuate students’ weakness ....................................................................................................... 56
  Change in the status of mistakes ................................................................................................... 57
  Adduce features ............................................................................................................................. 58
  Establishing ideas .......................................................................................................................... 60
  Raising attention ........................................................................................................................... 62
Learning attitudes ........................................................................................................................... 65
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exploratory, interactive and collaborative styles</strong></td>
<td>65</td>
</tr>
<tr>
<td><strong>Work becomes attractive</strong></td>
<td>67</td>
</tr>
<tr>
<td><strong>Increasing engagement</strong></td>
<td>68</td>
</tr>
<tr>
<td><strong>Promoting pupil autonomy</strong></td>
<td>68</td>
</tr>
<tr>
<td><strong>Peers as resource</strong></td>
<td>70</td>
</tr>
<tr>
<td><strong>Chapter 5: Discussion</strong></td>
<td>71</td>
</tr>
<tr>
<td><strong>Limitations</strong></td>
<td>73</td>
</tr>
<tr>
<td><strong>Recommendations for further research</strong></td>
<td>74</td>
</tr>
<tr>
<td><strong>Epilogue</strong></td>
<td>75</td>
</tr>
<tr>
<td><strong>References</strong></td>
<td>76</td>
</tr>
<tr>
<td><strong>Appendix A: Post-lesson interview protocol (translated)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Appendix B: Final interview protocol (translated)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Appendix C: Questionnaires protocol (translated)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Appendix D: Application for license from the Cypriot committee of primary education (Greek)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Appendix E: Application for license from the teacher and the headteacher of the school (Greek)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Appendix F: Application for license from students’ parents (translated)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Appendix G: Working sheets used in the lessons (Greek)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Appendix H: Transcriptions of classroom observations (Greek)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Appendix I: Transcriptions of interviews (Greek)</strong></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 1: Introduction

The societies in which we live have undergone rapid and widespread technological change in recent years. Information and communication technology (ICT\(^1\)) permeates our whole life including work, learning, leisure, relationships etc. Digital literacy will, if it does not already, undoubtedly play a significant role in our future lives (Allen, 2007).

Students nowadays live in a world where ICT plays a central role in their daily lives. They enter the classroom not only having encountered rich digital experiences but also being part of a society influenced by new technologies. In order for them to succeed in our digital culture, they need to be equipped not only with basic but also higher-order skills. Papert (1994) states that ‘not very long ago, young people would learn skills they could use in their work throughout life. Today, in industrial countries, most people are doing jobs that did not exist when they were born. The most important skill determining a person’s life pattern has already become the ability to learn new skills, to deal with the unexpected’ (p. vii). Education need to prepare students for their adult lives in today’s and tomorrow’s world, so that they can contribute in activities not as passive but as active and empowered participants (Pachler, 2001).

Undoubtedly we are empowered by technology and the challenge for education is to develop those human talents that technological tools cannot provide. With emerging information and communication technologies, the pressure has made everyone involved in the teaching process shift their views on effective teaching and learning even further. ICT and especially computers are considered to be necessary tools in classrooms and their use is mentioned in several of the goals of many National Curricula. Davis (2001) argues that ICT can play many roles in education that will continue to develop: ICT aspects of core skills, ICT as a theme of knowledge and ICT as a means of enriching learning.

New applications of technology have the potential to support learning across the curriculum and allow effective communication between teachers and learners in ways that have not been

---

\(^1\) In an educational context the acronym ICT stands for ‘Information and Communication Technology’ and refers to (a) the technological equipment available for educational use, (b) associated skills that students and teachers have to acquire and (c) a separate subject in many national curricula.
possible before. ICT has ‘the potential not only to support the current curriculum but also to enhance the experience and understanding of that curriculum and even extend thinking and learning in new ways’ (Loveless, 1995, p. 6). Students are provided with a sense of mastery over their environment; they are thinking about their thinking, checking their work and reflecting. The use of ICT promotes initiative and independent learning, with pupils being able to make informed judgments and develop the ability to be critical in their choices (DfEE, 1999). Loveless (1995) states that,

*ICT has the potential to organize and process information, freeing the children to ask questions, look for answers, take risks in exploration and use a wide range of resources for information. They can develop a positive attitude to their work by using real and relevant data and presenting work in a polished and accessible form. A positive experience of ICT in the classroom, developing children’s confidence and confidence in working as individuals and with others, should contribute to the general quality of their learning* (p. 120).

Teaching with ICT in the classroom is seen as qualitatively different from explicit, traditional teaching. While the need for effective use of ICT in teaching subjects across the curriculum is increasing, good practice remains uncommon (Ofsted, 2001) especially in Cyprus (Karagiorgi, 2005). Papert (1980) used a story that illustrates perfectly this situation: a group of time-traveller teachers and surgeons comes to our time from the past. Both are curious to see how their vocations have changed after all of this time. The surgeons visit a surgery; they recognize that they are in a surgery and that what they see being performed is an operation but they do not recognize most of the equipment they see. In contrast, the teachers visit a classroom and recognize that they are in a classroom and that what they see is teaching while they also recognize many of the teaching means.

Personally, I find it important for the Cypriot mathematical education not only to implement ICT resources in the classroom but also to use them effectively. Cyprus cannot and should not avoid the challenge of placing ICT into its educational agenda as new technologies are promoted. Education in Cyprus cannot ignore the impact of ICT in the social, cultural, economical and educational part of our lives. What can be said is that a school in the modern world without ICT
is almost as disadvantaged as a school without books (Way and Beardon, 2003). Mathematics need to change continually to reflect the needs of society thus educators need new ways of thinking about the development of mathematical knowledge.

In this sense and for these reasons, this work seeks to research the development of mathematical ideas and concepts through computer based teaching. The main aim is to analyze the role of a dynamic mathematics computer software called GeoGebra, as a tool in the teaching and learning of mathematics in the Cypriot primary school, by exploring its potential and the implications of its implementation in primary mathematics.

GeoGebra is a relatively new software system that integrates possibilities of both dynamic geometry and computer algebra in one tool for mathematics education. It allows a closer connection between the symbolic manipulation and visualization capabilities and dynamic changeability (Hohenwarter and Fuchs, 2004). Introducing GeoGebra in mathematics classrooms can be a way of providing opportunities for mathematical investigation, encouraging discussion and group work and generally it can make mathematics a more open and practical subject, which is accessible and manageable to more pupils (Hohenwarter and Fuchs, 2004).

There has been little work concerned with investigating whether children’s ideas about mathematics are affected by experience with GeoGebra and generally about its potential for teaching primary mathematics. Since not much has been written about this topic I set out to explore GeoGebra’s potential and implications in primary classroom practice. This exploration and understanding can only be established by carefully researching the world of the population that it concerns, that is, the primary classroom. This exploratory study seeks to listen to participants and see their perspectives on the topic, thus building an understanding based on their ideas and getting a complex, detailed understanding of the issue.
Chapter 2: Literature review

In recent years, with the increasing importance of new technologies for everyday life, educational technology\(^2\) has become a cornerstone for government efforts. Governments provide technology to schools and promote the use of ICT in schools across the curriculum in order to improve students’ performances (Wenglinsky, 1998). Bringing ICT into the classroom provokes innovation and change; in the absence of these fundamental changes to the teaching process, schools may do little but speed up ineffective processes and methods of teaching (Leidner and Jarvenpaa, 1995).

**Potentials of ICT for mathematics education**

The question arises here concerns the impact of ICT in mathematics education. This can be answered by exploring how technological changes interact with learning. Modern technology can provide students with a new means to experience mathematical concepts; it is essential for everyone involved in the teaching community to understand if these means affect and how they affect what students learn. Educators need to know the realities and the possibilities for learning in the era of ICT.

Several educational organizations have started to develop technology-related standards. In the US the National Council of Teachers of Mathematics (NCTM) considers technology as one of their six principles for school mathematics: ‘Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning’ (NCTM, 2000, p. 11). In England the Teacher Training Agency (TTA) offers a rationale for making use of ICT to support children’s learning of mathematics. They suggest that ICT has the potential to make a significant contribution to their pupil’s learning mathematics, because it can help them to:

- practise and consolidate number skills;
- explore, describe and explain number patterns;

---

\(^2\) Educational technology generally refers to the introduction of computers and other technologies to the classroom (Wenglinsky, 1998).
- take their first steps in mathematical modelling by exploring, interpreting and explaining patterns in data;
- experiment with and discuss patterns in number and shape and space;
- develop logical thinking and learn from immediate feedback;
- make connections within and across areas of mathematics;
- develop mental imagery and
- write simple procedures (TTA, 1999).

Additionally, the Qualifications and Curriculum Authority (QCA) states that:

*A sound grasp of ICT is essential in modern society; it gives pupils the skills and understanding needed to use technology effectively, every day and in the world of work ahead. Moreover, a sound grasp of ICT is fundamental to engagement in modern society; it teaches pupils how to find information appropriate to a task and to judge the accuracy and reliability of what they find. It gets pupils questioning and learning things for themselves and provides a gateway to information and experiences from a wide range of people, communities and cultures (QCA, 1998).*

One of the most interesting research fields in mathematics education concerns how to help students come to a ‘proper’ understanding of mathematics. A great number of teachers and researchers these days try to discover the impact of technology on teaching and learning of mathematics. The British Educational Communications and Technology Agency (BECTa, 2007b) argues that technology ‘improves attainment and helps raise standards, supports school improvement and efficiency, strengthens local authority data management and helps to personalize learning’. The use of ICT, wherever it is possible in the classroom, makes the teaching process more efficient and strengthens knowledge; there are claims that ICT has the potential to enhance cognitive learning, develop problem-solving and higher-level thinking skills and extend physical and mental abilities (Loveless, 1995).

Working with technology contributes to the students’ use of their mathematical knowledge and stimulates them into making their thinking visible and constructing their own knowledge (Hurme and Jarvela, 2005). Researchers have found evidence of a positive relationship between ICT use and educational attainment (BECTa, 2001). ICT can develop children’s knowledge,
understanding and skills concerning the following factors: finding things out, developing ideas and making things happen, exchanging and sharing information, reviewing, modifying and evaluating work as it progresses (Allen, 2007).

Evidence from research on the impact of ICT on intermediate outcomes, such as motivation, engagement and independence in learning, is increasing and more persuasive. The literature, especially in England, is very positive and rarely negative (Higgins, 1999) about aspects of ICT use. Many researchers have shown that in primary schools, the use of ICT by teachers is very effective in raising pupils’ motivation and extending their communication skills (BECTa, 2007a; DfES, 2003; OFSTED, 2005). This motivating power can be particularly effective for pupils with special educational needs (SEN). ICT can help them to overcome some of their barriers, for example their ability to produce legible and tidy work, and hence can raise their achievement (BECTa, 2007c).

The benefits gained from the use of ICT apply to all students, especially to students that have special educational needs (Franklin, 2001; OFSTED, 2004). Students who are reluctant learners or pupils for whom the classroom language is their second language or pupils with learning difficulties or disabilities, can work in private at their own pace without feeling that they are holding back. There is a substantial body of research into the ways in which ICT can support pupils with additional or special educational needs; ICT is a powerful tool in supporting inclusive practice (BECTa, 2007c). With ICT, students who have special educational needs have the right to access the whole curriculum; ICT facilitates both mixed abilities classrooms and inclusion education3 (Smith, 1999). The DES (1990) argues that ‘information technology is making a unique and valuable contribution to the learning of pupils with special educational needs, enriching their learning experiences and enhancing their access to a broad curriculum’ (p. vii). ICT is able to provide all children with access to communication, expression and information and thus a broader curriculum and experience (Loveless, 1995).

Wenglinsky (1998) refers to the debate on technology’s effectiveness. On the one hand advocates for technology assert that most uses of technology are valuable and can lead to

---

3 Inclusive learning is a process of increasing the presence, participation and achievement of all learners (BECTa, 2007c)
improvements; technology can support higher-order skills and increases students’ motivation. On the other hand, those who are opposed to technology assert that computers limit opportunities for social interaction and that the gains to academic achievement are not balanced to the costs of buying and maintaining technology. The use of ICT in the classrooms has caused the fear of social isolation or reduction of students’ social skills (Hennessy et al., 1989). Thompson (2003) argues that ICT promotes discussion and helps students to develop their thinking and understanding, particularly their mathematics thinking and their individual reasoning.

If educators accept that there are social, economic, intellectual and pedagogical reasons for using ICT in education, they need to consider not only how to use a range of ICT resources but also why and when to use them. If teachers do not understand the purpose for using such applications and the right time to use them then they may not get the innovations and changes they hoped for. In England, the National Numeracy Strategy (NNS) states that the teachers should use ICT in their daily mathematics lesson only if it is the most efficient and effective way to meet their lesson’s objectives (DFEE, 1999).

The fact that a particular technology is available in a classroom does not automatically mean that it will be used at all or that it will be used in a particular way or that it will have positive outcomes. Agalianos et al. (2001) argue that technologies and their use in the classroom are ‘socially contextualized and socially shaped’ (p. 1). Technology does matter to academic achievement but is dependant on how it is used. When used properly technology can lead to positive outcomes. It is important that technology is used in those areas where it provides benefits and reduced in areas where it does not (Wenglinsky, 1998).

**Integrating technology into mathematics teaching**

_In the first half of the twentieth century, mathematics was viewed as a set of procedures and principles that had to be taught before any potential mathematical understanding could take place... More recently this narrow view of mathematics has been superseded by a more progressive view of mathematical understanding that describes mathematics as being part of everyday life, wrapped up in culture and social practices (Bottle, 2005, p. 6)._
Teaching and learning mathematics should be an enjoyable experience for all students and ICT is an integral part of that enjoyment. Teachers should not deny students exciting and challenging experiences in mathematics and teach it only as a utilitarian subject. The use of information technology is a major current area of development in mathematics education. The first wave of technology was the pocket calculator and then microcomputers. Computers were first used in the classroom as aids in numerical calculation; however their most attractive development in the past years has been their potential to create and manipulate graphic images (Fey, 1989). Logo was one of the first protagonists in computer software for mathematics education. It presented a turtle robot that followed verbal mathematical instructions (Straesser, 2001).

In Cyprus, efforts to integrate ICT into education aimed at ICT as a cross-curricular tool and involved a combination of centralized initiatives and largely decentralized implementation responsibilities (Karagiorgi and Charalambous, 2004). In primary schools, the Ministry of Education and Culture made an effort to implement the humanistic curriculum model of ICT by emphasizing ‘using the machine in learning’ rather than ‘learning about the machine’. That means that the emphasis is on the student as a constructor of knowledge, on flexible pathways through curriculum areas and the use of computational media to facilitate learning, rather than on equipping the learners with skills that will help them in their vocational future as in a technocentric curriculum model (Nicholson, 1995).

**Why integrate ICT in mathematics education?**

Oldknow and Taylor (2000) argue that there are at least three reasons for integrating ICT in mathematics teaching in schools, namely, desirability, inevitability and public policy. Desirability can be supported in terms of students, teachers and schools; students are motivated, stimulated and encouraged; teachers improve their efficiency, are less administrative, allow more time for student-work and gain better records of their students’ progress; schools improve efficiency, educational inclusion and multilingual classrooms. In addition technology becomes inevitable at the time when conventional alternatives no longer exist (Oldknow and Taylor, 2000) and when its cost has been reduced to affordable amounts. As far as public policy in many countries there has been an acceptance of the educational benefits of ICT and thus governments
promote its use wherever and whenever it is possible starting from the first grade of compulsory education.

There are four reasons for incorporating ICT in education: speed and automatic functions, capacity and range, provisionality and interactivity (Loveless and Dore, 2002). Speed and automatic functions of ICT allow storing, changing and displaying information, analyzing and synthesizing information at higher levels leaving students time to think about the information presented. Capacity and range refers to the ability to access a vast amount of information that is distributed worldwide (Pachler, 2001). Provisionality enables users to make changes, try out alternatives, keep trace of their ideas and determine their own path (Allen, 2007; Loveless, 1995) whereas interactivity can engage students at a number of levels.

Technology is integrated into mathematics teaching and learning in two forms (Preiner, 2008). First, there are virtual manipulatives which consist of specific interactive learning environments. In the virtual manipulatives settings students can explore mathematical concepts without having special computer skills or knowledge about specific educational software packages. Secondly, there are mathematical software tools that are appropriate for educational purposes and can be used for a wide variety of mathematical content topics, thus allowing more flexibility and enabling both teachers and students to explore mathematical concepts.

*Teacher’s role*

Teachers nowadays have to incorporate various technologies into their teaching of mathematics. They have to translate into practice the high expectations and the visions of the supporters of technology while having students enter the classrooms having already acquired many ICT skills. Teachers should support students developing these skills and provide attractive opportunities to enrich learning. They also need to understand the ways to use information and communication technology in their classrooms in order to support and enhance their teaching and look to new technologies for better ways of reaching established goals (Allen, 2007). ICT provides the teacher with a wealth of tools to use in the classroom either for presentation and demonstration or for students’ group or individual work. Computer applications enable teachers to do things which are difficult or impossible to do in other ways (OFSTED, 2004).
As seen above, evidence from research show that there is a positive impact of ICT on attainment, motivation and learning; these benefits depend critically, however, on the decisions of teachers regarding how it is used. There is a strong relationship between the ways in which ICT is used and the outcomes, showing that teachers’ pedagogical approaches are crucial (DfES, 2002a). The way ICT is used in lessons is influenced by the teachers’ knowledge and beliefs about their subject and how ICT is related to it (Askew, 1997; BECTa, 2003). It is true that the extent of ICT use in the classroom is dependant on the individual teacher. NCTM (2000) states that:

*The effective use of technology in the mathematics classroom depends on the teacher.*
*Technology is not a panacea. As with any teaching tool, it can be used well or poorly. Teachers should use technology to enhance their students’ learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well — graphing, visualizing, and computing (p. 25).*

A lot of responsibility rests with teachers; choosing when and how to use these technologies in mathematics teaching is difficult. There is a wide variety of ICT that can be used in the teaching and learning of mathematics. Teachers should consider what ideas or topics ICT can best support, which aspect of ICT is most likely to support the topic being taught and how students are to be grouped in order to receive the most (Bottle, 2005; Thompson, 2003). It is the responsibility of the teacher to decide when technology can effectively improve learning opportunities and which kind of technology is appropriate to reach the objectives of the lesson.

Using computers and learning how to work with certain software is definitely a challenge for teachers, especially if they have no experience with new technology. Teachers not only need adequate training for all this but also time to accept and adapt to the changes into their classrooms (Laborde, 2001). Appropriate technology-related professional development needs to be provided in order to support teachers with this task, allowing them to effectively integrate ICT into their everyday teaching and introducing them to new software tools. Effective teachers who use ICT are teachers who are confident with ICT. They are much more comfortable with ICT as an ‘enabling addition to the pedagogical armory’ (Bramald *et al.*, 2000, p. 5) and are ready to let go of their control on the students (Agalianos *et al.*, 2001).
During the last two decades researchers have become increasingly aware of the role of teachers in an ICT classroom. Changing the emphasis in the mathematics classroom to a student-centered approach to learning involves a change in the teacher’s role (Hoyle and Sutherland, 1989). Many people claim that the computer will eventually replace the teacher; research however supports a very different conclusion, one which places the teacher at the centre of the educational process (O’Neill, 1998). It is difficult for machines to replace the personal relationship that exists between teacher and pupil in primary school, though the addition of new technologies may enhance this teaching and learning relationship (Straesser, 2001). Teachers’ roles are still reflected in the ways in which they organize and manage experience and resources in the classroom (Loveless, 1995). It is still true that excellent teaching and effective learning can only occur when a good teacher is present (Williams and Easingwood, 2004). Moreover, the role of the teacher is broadened beyond the classroom and the school in the forms of virtual learning, web-based discussion forums and computer conferencing systems (Selinger, 2001). The NCTM (2000) states that:

*Although students often can work independently of the teacher when using computers and appropriate software, new technology never will be able to replace teachers, since they play vital roles in technology-rich classroom settings. Thereby, technology potentially opens up new observation possibilities for teachers, allowing them to focus on students’ investigations and thinking strategies while solving mathematical problems* (p. 25).

The role of the teacher is not redundant, it changes (Selinger, 2001). The shift from an industrial society to a digital one requires educators to identify the implications for education and the skills pupils will need to cope in an increasingly transforming world (Younie, 2001). In today’s education teachers should not be seen as information providers but as learners who can engage in high level questioning, explaining, challenging and facilitating. The idea that computers could eventually replace teachers depends on a naive understanding of what is involved in the process of teaching and learning and a limited concept of the nature of knowledge (Fox et al., 2000).

The illusion that it is possible for schools to escape new technologies is losing ground (Younie, 2001). The implementation of ICT in schools does not only create new techniques in teaching the
existing curriculum more efficiently but can also change the content of the curriculum (Thompson, 2003). Using ICT effectively in schools is about more than changing resources; it is about changing practices and culture (BECTa, 2007b).

**Means of ICT in the classroom**

The incorporation of ICT in the classroom can involve computers, graphic calculators, cameras, handheld organizers, interactive whiteboards, digital video, digital camera, virtual learning environments, the internet, music, sound and much more. However, this study focuses on the use of computers in mathematics teaching and learning.

*Using computers in classrooms*

During the last 25 years, computer technology for mathematics classrooms has experienced an explosive growth both in terms of development as well as availability (Light and Blaye, 1989). The entrance of computers into the life of schools is undeniable (Wenglinsky, 1998). Substantial money has been invested by governments into equipping schools with hardware, software, and Internet access. The mathematics teacher’s book⁴ (MOEC, 1999) in Cyprus states that students should start using computers in the classroom as soon as possible. Giving students the opportunity to use ICT in the classroom helps them develop a positive attitude for mathematics, since they no longer view mathematics as ‘plain number activities’.

The long history of applying computers to mathematics learning began with the drill-and-practice programs and computer-assisted instruction (CAI) (Kaput and Thompson, 1994). Initially drill-and-practice programs were the most commonly available types of software to be used in schools (Finlayson and Cook, 1998). Later on, learning environments incorporating Logo were innovative in their use of technology as a cognitive tool in mathematical thinking and provided students opportunities for knowledge construction (De Corte, 1996). Logo was based

---

⁴ The mathematics teacher’s book is a guidance book for teaching mathematics that includes lesson’s goals and proposes activities. It is published by the Ministry of Education and Culture in Cyprus and it is distributed to all teachers.
on the idea of people constructing knowledge from their experience and the notion that the teacher should be a facilitator rather than an instructor (Agalianos et al., 2001).

Computers are a powerful means of doing mathematics. They can be used to reorganize both the individual mental life of students (and teachers) and the broader context of the educational environment (Pea, 1985). The interactive environment of computer-based learning is exciting and different (Hoyles and Sutherland, 1989). What happens on the screen of a computer can be more subtle and interesting than what happens on a chalkboard (Fox et al., 2000).

There are three levels at which computers can be used in the daily mathematics lesson; these are the class level (whole-class discussion and review) the group level and the individual level. An effective use of computers in classrooms can provide the environment that many teachers long for while they struggle with the constraints of the traditional classroom (Fey, 1989). Moreover, computer use has the potential to free teachers from the moment-by-moment demands of whole-class teaching providing them an environment of peer support and collaborative work (Mcdonald and Ingvarson, 1997).

Computers have two central functions; they can be used as tools in mathematics to perform routine processes or to explore mathematical ideas (Williams and Easingwood, 2004). The National Council for Education Technology (NCET, now the BECTa) defines five major opportunities which the use of computers is able to provide in order to support learning in mathematics (NCET, 1997):

- Learning from feedback: this refers to the ability computers have to give feedback to the users, which comprises a fundamental part of the learning process. The feedback given is informative rather than evaluative; computers evaluate students’ actions but do not judge them (Hoyles, 1991). The feedback that is available when using a variety of programs, along with the facility to make changes easily, encourages the children to conjecture, hypothesize and try out ideas (Briggs and Pritchard, 2002). Depending on the type of feedback students receive, they engage in different learning levels; if a software provides feedback according to the number of correct responses or the total score then the students are less likely to correct their errors whereas if they are offered formative feedback it helps students identify what
they can improve and students are more likely to correct their errors and learn from them (Briggs and Pritchard, 2002).

- Observation of patterns: computers offer opportunities to generate patterns faster allowing more time to concentrate on these patterns (Briggs and Pritchard, 2002). Computer programs can either be specifically written for the purpose of observing patterns or can be set up for this purpose (Briggs and Pritchard, 2002).

- Computers can store, organize, manipulate and present large amounts of data quickly, whereas without technology it is reasonable, to keep the numbers simple and the number of examples to minimum (Loveless, 1995).

- Teaching the computer: this refers to the process of giving instructions to be performed by the computer which is an empowering skill that help students go beyond any constraints of the software (Fey, 1989). The need to teach the computer to carry out particular tasks leads children to develop their thinking and wonder about shapes, arithmetic, algebra and sequence (Briggs and Pritchard, 2002). It also has the potential to develop good mental habits and help in various aspects of learning mathematics (Fey, 1989).

- Develop visual imagery: this is a fundamental aspect of mathematical reasoning. It is likely that students seeing something happen dynamically will come to a deeper understanding of the shapes and their relationships to each other than if they had only experienced a static version of the same phenomena (Briggs and Pritchard, 2002). Apart from this dynamic function students can see multi-representations, including symbolic, analytic, numerical and visual representations where each mode strengthens the understanding of the other (Elliott et al., 2000; Fey, 1989).

Computers are not just another tool, they have the potential to fundamentally alter the way teachers teach (Franklin, 2001). However, placing students in front of a computer will not mean that they automatically learn more. The power of ICT is not automatically used every time a computer is switched on. Teachers must establish a learning environment where this can happen. They should ensure that the computers in the classroom are used in such way that they benefit teaching and learning of mathematics and provide opportunities which would not be available
without the use of computers. Hoyles and Sutherland (1989) claim that ‘the computer is a new powerful classroom resource and unless teachers treat it as such and modify their roles accordingly there will be little advantage for pupil learning from its use’ (p. 141).

**Types of software**

More than ten years ago when computers first began to appear in primary school classrooms, there seemed to be an unspoken belief that there was a strong and undeniable link between computers and mathematics. Not surprisingly the link was made; a lot of software with mathematics topics was introduced and a lot of technological tools were used in the classrooms thus reinforcing the link between ICT-supported teaching and mathematics.

Computer software can support children’s discussion while giving them a chance to work independently of the teacher (Wegerif and Dawes, 1998). With the use of carefully-developed software, computers have the potential to become a significant contributor to classrooms (Hennessy *et al.*, 1989). Papert (1973) identifies three features of educational software: it gives agency to the machine not the child, it is deceptive and it favours quick relations over long-term thinking.

Teachers can choose from a variety of software available for mathematics education. These types of software are derived from different educational paradigms and have different positions about the pedagogy in a school class (Hoyles, 1991). What teachers should take into considerations are pedagogical, mathematical and organizational aspects of the software (Oldknow and Taylor, 2000). Pedagogical issues relate to the capability of the software to help teaching, enhance knowledge and practice, strengthen skills and improve understanding. Mathematical issues relate to the mathematical abilities of the software. Organizational issues refer to the capability of the software to manage time, produce materials more efficiently and to find resources. Returning to the variety of available software, teachers can choose between:

- small software; programs that focus on specific curriculum content;

---

5 There is also a wide range of software for SEN (Allen, 2007).
- programming languages, e.g. Logo, Basic;
- programming graphing calculators, e.g. Texas instrument TI-83;
- generic software, e.g. spreadsheets and databases;
- content-free, subject-specific software, e.g. Cabri, Sketchpad;
- graph-plotting software;
- computer algebra systems (CAD);
- dynamic geometry software (DGS), e.g. Cabri-Géomètre and ‘The geometer’s Sketchpad’;
- data-handling software (DHS);
- mathematical communication tools (MCT);
- graphing calculators;
- data-loggers, e.g. CBR and CBL and
- CD-ROM and the Internet as sources of data (Oldknow and Taylor, 2000).

Computer software is designed with different purposes in mind. There are different methods of software classification. One simple system of classification is that a computer program can either be a tutor, a simulator or a tool (Briggs and Pritchard, 2002). Tutor software is designed to be used repetitively and can resemble a textbook where many examples are given for practice. This tutorial use of computers delivers knowledge to the learner who can have a minimum degree of control. The term drill-and-practice is sometimes used for this type of software. It can take a behaviourist approach by rewarding a correct response or punishing a wrong one or a constructivist approach in which the user is encouraged to investigate. These programs do not have the potential to transform teaching and learning but can do the same things that schools have traditionally done perhaps more systematically and efficiently (Briggs and Pritchard, 2002).

A simulator puts the user in a different imaginary context or in a construction of a real situation. The user tries to discover the variables and rules by trial and error and feedback. Tool software
allows certain jobs such as writing, drawing, communicating etc, to be carried out more easily (Briggs and Pritchard, 2002) allowing more time for the learner to be more productive in the analysis, interpretation and presentation of information.

Another classification sets out four subdivisions: instructional (similar to the tutor), revelatory (similar to simulations), conjectural (allows exploration and manipulation and testing of ideas which enable the learners to set up their own models and test hypothesis e.g. Logo) and emancipator (similar to a tool) (Kemmis et al., 1977). At a more fundamental level it is possible to classify most software as either content-free or as content-specific (Briggs and Pritchard, 2002). Content-free software, also referred as generic software, does not have specific content. It is possible to view content-free software as a tool with a specific or sometimes varied and versatile job that it can undertake, but in no particular subject domain. This kind of software is widely available and has the potential to support students’ learning in mathematics. Content-specific software is also very common and is specifically written to encourage the learning of particular mathematical topics.

An additional classification of software is the one between open and close-ended. Open-ended software allows the opportunity for its users to select from a wide choice of actions and each one to produce a different computer response. On the contrast, closed-ended software plays a more passive role; the level of computer contribution is limited to the organization and display of the information given by the user (Finlayson and Cook, 1998).

In conclusion, computer software can be used as a tool that enables students to work efficiently and present information clearly, as a resource for learning that provides support in teaching and learning and as a catalyst that helps students to think in new ways by conjecturing in order to explore their ideas and try out new possibilities. The balance of this work, however, is weighted towards the role of the computer as a tool for observing patterns, exploring data, problem solving and inquiry.
The Dynamic Mathematics Software GeoGebra

What is GeoGebra?

GeoGebra is a Dynamic Mathematics Software (DMS) for teaching and learning mathematics that combines many aspects of different mathematical packages (Hohenwarter and Lavicza, 2007). It is a form of freely-available, open-source educational mathematics software that provides a flexible tool for visualizing mathematical ideas from elementary to university level, ranging from simple to complex constructions (Hohenwarter and Jones, 2007). It dynamically joins geometry, algebra and calculus offering these features in a fully connected software environment (Hohenwarter and Lavicza, 2007). It is as easy to use as Dynamic Geometry Software (DGS) but also provides basic features of Computer Algebra Systems (CAS).

Why is GeoGebra different?

Atiyah (2001) refers to geometry and algebra as ‘the two formal pillars of mathematics’. GeoGebra is an attempt to join these pillars, which other packages treat separately, into a single package. The basic idea of GeoGebra is to provide a dynamic software that incorporates geometry, algebra, and calculus and treats them as equal partners thus enhancing the teaching of mathematics through enabling learners to gain stronger links between geometry and algebra (Hohenwarter and Jones, 2007; Hohenwarter and Lavicza, 2007).

The most notable feature of GeoGebra is that it offers two representations of every object: every expression in the algebra window corresponds to an object in the geometry window and vice versa.

---

6 Open source software (OSS) has been defined as ‘software for which the underlying programming code is available to the users so that they may read it, make changes to it, and build new versions of the software incorporating their changes’ (BECTa, 2005, p. 2).
7 Dynamic geometry software (DGS) is a type of software which allows one to create and then manipulate geometric constructions. The contribution of DGS is two-fold: first it provides an environment in which students can experiment freely and second, it provides non-traditional ways for students to learn and understand mathematical concepts and methods (Jones, 1997; Marrades and Gutierrez, 2000).
8 Computer algebra systems (CAS) are designed to facilitate the manipulations of mathematical expressions in symbolic form. CAS can contribute to the development of mathematical knowledge because developing graphic and symbolic reasoning using CAS influences the range and form of the tasks and techniques experienced by students. Moreover using CAS results in the development of mathematical knowledge, as a means of facilitating and extending experimentation with mathematical systems (Ruthven, 2002).
versa providing a deeper insight in the relations between geometry and algebra (figure 2.1). GeoGebra provides the facility to move between the algebra window and the geometry window. On the one hand, the geometric representation can be modified by dragging it with the mouse like in any other dynamic geometry system, whereby the algebraic representation is changed dynamically. On the other hand, the algebraic representation can be changed using the keyboard causing GeoGebra to automatically adjust the related geometric representation.

What has GeoGebra to offer?

As a software package that combines both geometry and algebra, GeoGebra has much to offer (Hohenwarter and Jones, 2007). Geogebra is specifically designed for educational purposes and can help students to foster their mathematical learning (Hohenwarter and Preiner, 2007). Its environment is mathematically rich and due to the fact that it is interactive it promotes mathematical explorations. It also provides a wide range of mathematical concepts which are dynamic and thus more accessible to pupils. Geogebra provides a visual and conceptual feedback to the learner. In addition, it is free, so pupils can use it not only at school but also at home thus they have the opportunity to do their homework, practice, revise the lesson and prepare for the next one. It is available in a range of languages offering a great opportunity to use the software in local languages and in multicultural classroom environments.

The software includes a geometry window, a toolbar (figure 2.2), an algebra window, an input field, a menu-bar and construction protocol and a navigation bar (figure 2.1). The construction protocol offers the researcher and the teacher a step-by-step record of the pupils’ computer interaction, which represents an important part of the pupils’ choices and actions. Thus it enables them to obtain a relatively precise image of the strategies used by pupils to solve a given problem.

The open source nature of Geogebra has encouraged a worldwide communication between its users. They can have access to GeoGebraWiki, a pool of materials that allows everyone to contribute their own creations or take an existing worksheet and produce a customized version. They can also access the GeoGebra UserForum where they can discuss their questions and ideas (Hohenwarter and Preiner, 2007).
Figure 2.1: Screenshot from a Geogebra window.

**Teaching Mathematics with GeoGebra**

Skills, pedagogy and curriculum are the three aspects involved in the use of Geogebra in the classroom. Teachers need to know how it works and how it can be effectively integrated both within the classroom and within the curriculum. Thus, when incorporating Geogebra in the classroom these fundamental features should be taken in mind.

Geogebra can be used in many ways in the teaching and learning of mathematics: for demonstration and visualization since it can provide different representations; as a construction tool since it has the abilities for constructing shapes; for investigation to discover mathematics since it can help to create a suitable atmosphere for learning; and for preparing teaching materials using it as a cooperation, communication and representation tool (Hohenwarter and Fuchs, 2004).

The success of GeoGebra has shown that non-commercial software packages have the potential to influence mathematics teaching and learning worldwide (Hohenwarter and Lavicza, 2007) without governments having to invest a tidy sum of money in supplying schools with software.
Figure 2.2: List of all Geogebra tools. Next to each tool there are the name of the tool and guidelines for the user in order to construct each tool e.g. to construct a parallel line one have to click on a point and then on the parallel line.
Research questions

This work seeks to research the development of mathematical ideas and concepts through computer based teaching. The main aim is to analyze the role of Geogebra, as a pedagogical tool in the teaching and learning of mathematics in Cypriot primary schools, by exploring its potential and the implications of its implementation in primary mathematics. More specifically the research questions are:

i. What are the potentials of GeoGebra for mathematics teaching and learning?

ii. To what extend does the use of GeoGebra support a constructivist teaching approach?

iii. To what extent is GeoGebra manageable in primary classrooms?

iv. How are classroom dynamics shaped by the implementation of GeoGebra? (What roles does the teacher adopt, to what extend does it help the classroom activity?)

v. What are the students’ learning attitudes while using Geogebra?
Chapter 3: Research Design

This chapter describes the methodological rational for the study. There is a range of methodologies and methods available for researchers. In mathematics education researchers should make explicit the theories that influence their work since these theories influence both the ways in which they work in the classroom and the ways they analyze their data. The following research design is structured according to Crotty’s (1998) suggested research processes.

Crotty (1998) argues that in developing a research design the researcher should answer two basic questions: firstly, what methodologies and methods will be employed in the research and secondly how this choice and use of methods and methodologies is testified. The second question deals not only with the purpose of the research but also with the researcher’s understanding of reality (theoretical perspective) and about what human knowledge is and what it entails (epistemology) (Crotty, 1998). Thus the two initial questions have expanded into four: what epistemology is embedded in the theoretical perspective, what theoretical perspective lies behind the methodology, what methodology controls our choice and use of methods and what methods are proposed to be used. These four elements are presented separately because each element is substantially different from the other (Crotty, 1998).

Epistemology

Epistemology refers to the theory of knowledge embedded in the theoretical perspective (Crotty, 1998). This study is based on a social constructivist view of learning: pupils learn mathematics through active construction of their own knowledge and this can be facilitated in a computer environment through the interactive process of conjecture, feedback, critical thinking, discovery and collaboration (Howard et al., 1990). I consider constructivism to be the socially collective generation and construction of meanings rather than a meaning-making activity of the individual mind as Crotty (1998) claims. I do not take constructivism to highlight the unique experience of an individual that tends to resist the critical spirit (Crotty, 1998); in contrast, I ground my research on a social constructivist nature of knowledge in which:

The meanings are negotiated socially and historically. In other words they are not simply imprinted on individuals but are formed through interaction with others (hence
social constructivism) and through historical and cultural norms that operate in individuals’ lives (Creswell, 2003, p. 8).

Social constructivism claims that rather than being transmitted, knowledge is created or constructed by each learner (Leidner and Jarvenpaa, 1995); there is no knowledge independent of the meaning attributed to experience constructed by the learner (Hein, 1991). According to certain cognitive theories learning does not involve a passive reception of information, instead, the learning process can be regarded as an active construction of knowledge in a learner-centered instruction (Kapa, 1999). Constructivism claims that students cannot be given knowledge; students learn best when they discover things, build their own theories and try them out rather than when they are simply told or instructed. Vygotsky argues that

Direct teaching of concepts is impossible and fruitless. A teacher who tries to do this accomplishes nothing but empty verbalism, a parrot-like repetition of words by the child, simulating a knowledge of the corresponding concepts but actually covering up a vacuum (Vygotsky, 1962, p. 83).

By participating in social constructivism activity students have the opportunity not only to learn mathematical skills and procedures, but also to explain and justify their own thinking and discuss their observations (Silver, 1996). From a social constructivist perspective ICT offers teachers a powerful pedagogical tool-kit (O’Neill, 1998). Hoyles (1991) argues that in mathematics lessons involving computers, learning is achieved through social interaction for three reasons: the social nature of mathematics; the collaboration that computer based activities invite; the basis for viewing the computer as one of the partners of the discourse.

Theoretical perspective

The theoretical perspective refers the philosophical stance informing the methodology, providing a context for the process followed and justifying its logic (Crotty, 1998). Recognizing the fact that there are multiple socially constructed realities this study adopts the interpretive paradigm and more specifically symbolic interactionism as the primary theoretical perspective.

The interpretive paradigm emerged in the social sciences to break away from the constraints imposed by positivism (Boghossian, 2006). The main aspiration of the interpretive paradigm is
to understand ‘the subjective world of human experience’ (Cohen et al., 2007, p. 21). In order to maintain the integrity of the investigated phenomena efforts are made by the researchers to enter into the culture and find out the insider’s perspective. Interpretive researchers see reality as a social construct and try to understand individuals’ interpretations of the world around them (Bassey, 1992). Researchers work directly with experience and understanding in order to see the observer’s viewpoint and thus build the theory (Cohen et al., 2007).

As seen above, Creswell (2007) argues that the meanings constructed are negotiated socially and historically. The interpretivist approach looks for ‘culturally derived and historically situated interpretations of the social life’ (Creswell, 2007, p. 21) while symbolic interactionism ‘explores the understanding and meanings in culture as the meaningful matrix that guides our lives’ (Crotty, 1998, p. 71).

This is directly linked to the purpose of the research: to get inside the classroom and see how the computer software could be used as an aid in pupils’ understanding of mathematics. Only through significant symbols, for example language and other symbolic tools which humans within a culture share and use to communicate, researchers can become aware of the insiders’ perceptions and attitudes and interpret their meanings and intentions; hence symbolic interactionism (Cohen et al., 2007; Crotty, 1998).

**Methodology**

Methodology refers to the strategy, the process or the design behind the choice and use of particular methods (Crotty, 1998). The methodology followed in this research can be described as a case study. I choose to view case study as a methodology (Yin, 2003) in contrast to Stake (1995) who states that case study research is not a methodology but a choice of what is to be studied. Case study research is a methodology in which the investigator explores a bounded system within its real life context (the case or multiple cases can be a child, a school, a classroom, a community, etc.) over a sustained period of time through data collection involving multiple sources of information in order to gain an in-depth exploration and analyze systematically the manifold phenomena that constitute this system (Cohen et al., 2007; Creswell, 2007; Robson, 2002; Yin, 2003). It provides a unique example of real people in real situations, enabling readers to understand ideas more clearly and investigate the complex interactions of
events in the unique and dynamic context of the case embedded in its cultural context (Cohen et al., 2007; Verschuren, 2003).

I conducted a single instrumental case study: the research was focused on an issue and selected one bounded case to illustrate this issue (Creswell, 2007; Stake, 1995; Yin, 2003). Case study research provided me a way to study and describe the behaviour of individuals-students (something that is highly complex) within their social context (the classroom) over a period of time, allowing different methods of data collection to be incorporated. However, case study researchers are bound by time so this collection must be completed over a sustained period of time (Stake, 1995).

Moreover, constructivism helps a case study researcher to justify many narrative descriptions since the emphasis is on detailed description and interpretations of the people comprising the case (Stake, 1995). One of the characteristics of case studies and qualitative studies in general that is most criticized is the lack of external validity or generalizability because of the small number of research units (Verschuren, 2003). However, the aim of this study is not to generalize findings to schools across Cyprus but to obtain a deeper understanding of the implementation of Geogebra as experienced by the participants. This is possible through a case study methodology: Yin (2003) argues that the results of a case study are not generalizable to populations but can generalize theoretical propositions, and refers to these as statistical and analytical generalizations.

**Participants and procedure**

The research involved a teacher and her 6th grade class in a rural state primary school in Cyprus. The classroom consisted of 16 pupils -10 boys and 6 girls- aged from 11 to 12 years. Students were informed by their teacher that I came from an English university, and that I was doing research on the use of computers in mathematics and that for that research they would have three mathematics lessons on the computers.

The research followed various stages: sending the proposal of the research and getting a license from the Cypriot committee of primary education; identifying the case, that meant finding a primary school teacher who was willing to collaborate in the research; getting permission from
the headteacher of the school and the parents of the students involved; choosing a topic to teach; designing three lessons (with the teacher); developing a sequence of activities (with the teacher) taking into consideration the suggested activities from the ministry (since it was one of the conditions of the permission given); developing worksheets (with the teacher); conducting the lessons and carrying out the activities with the students. The learning objectives of the lessons were similar with the objectives set by the ministry for the particular lessons. The teacher taught the lessons which involved students’ exploration on the computer. The case study students were involved in classroom work and interactive group work without having any experience of GeoGebra prior to their involvement in the project. The structure, the activities and the worksheets of the lessons were developed in cooperation with the teacher. However, I took much of the responsibility for designing the activities in GeoGebra as the teacher had never worked with computers for mathematics lessons and had never used GeoGebra. During the work in the classroom, I was a participant observer. I helped the teacher managing the classroom when the students were working on the computers and I kept notes of the general classroom situation.

The lessons were conducted in an ICT suite. In this room there was space for other work resources such as paper, books and mathematics equipment and for the pupils to sit in pairs and take turns in using the keyboard and the mouse. Useful information about tools was provided through the projector to help students in case they could not find the tool they wanted to use. Students worked autonomously and engaged in collaborative learning and group work. Asking the students to work in pairs with the computer was not related to the limited ICT resources. There were many reasons that led to this decision. It is an opportunity to exercise cognitive and social development, flexibility and organization for the teacher, the ability to cooperate, discuss and negotiate with others. Sharan et al. (1981) argue that learning in small groups can improve achievement and mental functions. In establishing pairs, students chose their own partners and remained constant throughout the lessons. However, I asked the teacher to recommend two children to form a pair that would be videotaped (in the observations, it became apparent that these were two of the most able students in mathematics).

Another consideration was the extent to which the group would use the computer, if they would be able to make progress and work on the software without the constant supervision of the
teacher. GeoGebra, however, seemed particularly easy and intuitive to learn (Sangwin, 2007). In these three lessons students had to discover relationships between the radius, circumference, diameter and area of the circle. During the lessons, pupils did not only experience a presentation of GeoGebra as a demonstration tool but also interacted with our prepared applications and created their own files. The first lesson involved some basic definitions about the circle and the exploration of the relation between the radius and the diameter (figure 3.1). Although the relation between the radius and the diameter of the circle could be approached deductively, the teacher and I chose to do it empirically by students’ exploration in GeoGebra since we wanted students to interact with the software. However, after this initial experimental approach, the teacher and the students also proved it deductively during the classroom discussion.

![Diagram of circle with annotations](translated)

_Drag the point C and observe how the quotient of the division changes_

**Figure 3.1: Example of an application used in the first lesson (translated).**

The second lesson involved the exploration of the relation between the circumference and the diameter. The students discovered $\pi$ and were asked to calculate and then apply the formula of the circumference (figure 3.2).
In the third and final lesson, students investigated the relationship between the area of the circle and the radius and concluded with the formula of the area (figure 3.3). In these lessons students needed activities in which the proof was convincing and illustrative. The activities designed in GeoGebra tried to satisfy this condition.

Figure 3.2: Example of an application used in the second lesson (translated).

Figure 3.3: An application used in the third lesson. Stages 1-4 show what happens while moving the slider.
Both the cooperative and the individualized learning sessions included three parts: teacher introduction to the whole class (about 10 minutes), cooperative or individualized work (about 20-25 minutes), and teacher review with the whole class (about 5-10 minutes).

- **Introduction**: the key concept is introduced to the students. By the end of this phase the students and the teacher set up the problem.

- **Practical session**: students engage in activities on the computer in order to explore and solve the problem that came from the previous phase.

- **Plenary**: this is where the new material is discussed. Students discussed what they discovered and teacher tried to reinforce and extend students’ learning. The students’ results of their experiments with GeoGebra were the basis for discussions in class. Teacher’s intervention at this stage aimed to help students understand what they had discovered in order to make richer meanings.

During the introduction and the plenary, students were sitting in the center of the room at round desks participating in the whole-class discussion whereas in the practical session they moved to the computers investigating and exploring collaboratively in pairs.

*Pilot study*

Robson (2002) refers to the pilot study as a small-scale version of the real research. It is a trail of the proposed research to check its feasibility. Yin (2003) views pilot tests as helping the researchers to improve their data collection plans regarding the content of the data and the procedures to be followed.

To determine the suitability of the items, a pilot test was held in which the three lessons designed were taught informally. The participants were three students from another school who did not participated in the research, had never used Geogebra before, and were selected in a purposeful way. In the pilot test, which lasted three days, these three students were taught the lessons as they were designed. As a result of the pilot test, a number of changes to the research design were made.
Ethical issues

The majority of the students live in homes which are rich in ICT, using computers, CD-ROMs and the Internet regularly with help from parents and siblings. However, there are students who come from less affluent or less richly resourced backgrounds; all children that enter school do not have the same ICT skills. Thus some students with higher digital skills will use the software with more expertise than others. In order to limit that disparity, all students that participated in the research had a training lesson in GeoGebra.

As stated before, GeoGebra is freely-available, open-source software, thus, there isn’t any license issue in its use. However, the most important ethical issue in this research concerns confidentiality and anonymity. I assured the participants confidentiality of the data and protection of identities. Although there was a form of intrusion into the teaching practice I had everyone’s informed consent; from the committee of primary education, from the teacher and headteacher themselves and from the students’ parents. These people were provided with adequate information (a written statement that outlined the nature, purpose and procedure of the research, what data would be gathered and how it would be used) about the research and they all had the right to withdraw (see appendices D, E, F).

Methods

Methods refer to the techniques or procedures used to gather and analyze data related to the research questions (Crotty, 1998). In the research I used qualitative methods of collecting data: videotaped observations of the lessons, interviews from the teacher and questionnaires given to the students.

In this qualitative approach based on constructivist perspectives I collected open-ended emerging data with the primary intent of developing themes from the data (Creswell, 2003). In the model followed here the distinction between qualitative and quantitative research occurs at the level of methods. Crotty (1998) argues that the distinction between what Creswell (2003) calls approaches to research -qualitative and quantitative- occurs at the level of methods and not at any of the other levels.
In this study, I used several data collection methods to ensure triangulation of the data; many sources of data are better than a single source as multiple sources lead to a fuller understanding of the phenomena studied (Denscombe, 2007). Data collected in one way can be used to cross-check the accuracy of data gathered in another way (LeCompte et al., 1993). Triangulation also assists in correcting the bias that occurs from the fact that in this research there is only one observer of the phenomenon under investigation.

Each method reveals its own aspects and parts of social reality (Verschuren, 2003). For example observations show behaviour but not motivation whereas interviews show motivation but not behaviour (Cohen et al., 2007). I wanted to see multiple perspectives – teacher’s and students’ – in order to have a more holistic idea, and to avoid ‘tunnel vision’ (Verschuren, 2003). Thus, the interviews further clarified what had happened in the classroom, revealed the teacher’s perspective on the matter and provided a means of triangulating inferences made about the mathematical activity in the classroom. The questionnaires illustrated students’ perspective about the lessons described.

During the research, I maintained a journal recording my actions such as daily schedule, lesson plans and self-reflections on each lesson. The major sources of data were the videotapes from observations, students’ questionnaires and teacher’s interviews. Field notes taken during observation and my journal were supplementary sources.

Observations

Observation is a powerful tool that offers the researcher the chance to gather live data from the social situations as they emerge, getting inside situations and observing directly what is happening, thus collecting more valid and authentic data (Cohen et al., 2007). Simpson (2003) argues that there are several strengths in observations as a tool for gathering data; however, observations are prone to difficulties and weaknesses as well. On the one hand observations can give direct access to social interactions, can enrich data gathered by other techniques and can give permanent and systematic records of social interactions (Robson, 2002; Simpson, 2003). On the other hand observations have high demand on time, effort and resources and they are vulnerable to the observer’s bias (Simpson, 2003). I tried to reduce this bias by triangulating my data.
During the lessons video recordings were made of all of the GeoGebra sessions. Two cameras were placed in the classroom: one videotaping the whole classroom interaction and the other videotaping a pair of students who were working on one computer. Given this method of data collection, it was possible for me to move away from the computer and observe from a distance and when needed, help students who faced problems. It also freed me from the need to take details notes.

During the observation I chose to adopt the role of the observer as participant. Adopting this role was particular helpful for the data collection. Cohen et al., (2007) argue that participant observation is particularly useful in studying small groups for a short time and the data derived from it is ‘strong on reality’ (p. 405). Through participant observation, researchers have the chance to study the behaviours as they occur and to become friendlier with the participants so that they will not consider them an intruder (Simpson, 2003).

*Interviews*

*The use of the interviews in research shows a move away from seeing participants as simply manipulable and data as somehow external to individuals, and towards regarding knowledge as generated between humans often through conversations* (Kvale, 1996, p. 11).

Influenced by constructivism, knowledge is seen as constructed between humans, and interviews can serve as a significant tool in this process of construction. An interview can be regarded as a change of views between two or more people on a topic, enabling verbal, non-verbal, spoken and heard channels to be used (Cohen et al., 2007). Cannell and Kahn (1968) have defined the interview as a conversation between two people, which begins with the interviewer with the purpose of collecting data relevant to their research, and focuses on content which is determined by the research’s goals. However, an interview cannot be considered an ordinary conversation since it has a specific purpose, it is based on questions asked by the interviewer and the responses have to be as explicit as possible (Cohen et al., 2007).

I firstly informed the teacher about the nature and the purpose of the interview and then tried to establish an appropriate atmosphere so that the teacher could feel secure to talk freely. I also
informed her that the interview was going to be recorded. Four semi-structured interviews were designed: one interview after each lesson and one interview after the completion of all the lessons. I used semi-structured interviews since they are flexible allowing new questions to be brought up during the interview (Cohen et al., 2007; Drever, 2003) and that helped me gather the data I sought. In the interviews my aim was to see the teacher’s perspective about the lesson’s objectives, if they were achieved, and if the use of GeoGebra facilitated (a) the students and (b) the teaching. I also thought that it would be important to see if the teacher noted a significant moment that took place during the lesson. Therefore the interview included the following questions:

1. What were your intentions in this lesson?

2. Do you think that those intentions were achieved?

3. Do you think that students benefited from the use of GeoGebra in today’s lesson, or not? How do you support your answer?

4. Did the use of GeoGebra facilitate your teaching?

5. Was there a significant moment in today’s lesson that you would like to mention?

The first question aimed to make the teacher feel more comfortable and served as a warm-up stage since the learning objectives were already known by the researcher. In the interview I wanted to see the teacher’s perspective about the use of GeoGebra in mathematics. I wanted to gather information about positive and negative aspects concerning the use of the software that she witnessed during the lessons; how the lessons conducted differ from the traditional mathematical lesson; if it was easy for the teacher to integrate the software into her teaching; if GeoGebra contributed in achieving the lessons’ objectives and to what extend these objectives would be achieved if she had not use it. Thus the final interview included the following questions:

1. Does a lesson with the use of GeoGebra differ from a typical mathematical lesson? If yes then how? What changed?

2. How easy was it for you to incorporate GeoGebra into your teaching?
3. Did GeoGebra contribute in achieving the learning goals set in these three lessons?

4. To what extent would the learning goals be achieved if you did not use GeoGebra?

5. Finally, I would like you to mention some advantages and disadvantages you noticed regarding the use of GeoGebra in mathematics.

Some of the questions used could be answered with a simple yes or no. Knowing this fact I had some probing questions in mind that would get the interview to expand their response. I used open-ended questions during the interview since I did not want the interviewee to be constrained in her answers. Open-ended questions are flexible, show the limits of the interviewee’s knowledge, encourage cooperation between the interviewer and the interviewee, allow the interviewer to have a more informed account of what the interviewee supports and can produce unexpected answers (Cohen et al., 2007; Drever, 2003; Robson, 2002). Creswell (2003) argues that the more open-ended the questioning the better as the researcher listens carefully to what people say.

In order to ensure the validity and feasibility of my questions I did a pilot study with a colleague of mine. I put her in an imaginary situation -the same situation in which the teacher would find herself after the lessons- and asked her the same questions. I compared the content of the ‘answers’ my colleague gave me with the content I was expecting to receive. Through this procedure I improved my questions in order to be more valid. However, validity does not imply reliability; the consistency of the teacher’s answers would reveal the levels of reliability of the questions.

**Questionnaires**

Apart from classroom observations and interviews with the teacher, short anonymous questionnaires were given to the students in order to see their perspective on the lessons conducted. I gave questionnaires to the students because at this age a questionnaire can be more reliable than other methods of collecting data. A questionnaire is anonymous, encourages honesty and it is more economic in terms of time (Cohen et al., 2007). In order to ensure the validity and reliability of the questionnaire, I did a pilot study of the questionnaire. The aim of the questionnaire pilot study was to simulate the real thing as closely as possible by using a
similar population and setting up the same conditions for administration and response in order to see roughly how long it takes to answer the questionnaire and if the questions are clear or need further explanations (Munn and Drever, 2004). The pilot study of the questionnaire helped me to make the language and syntax of the questions less complex in order for the questionnaire to be more clear, comprehensible, reliable and valid.

The questionnaires were given to the students one day after the completion of the three lessons; they were brief, easy to understand and reasonably quick to complete. They included a small introduction followed by three open-ended questions:

During the last three days we used computers in our mathematics lessons, and in particular, Geogebra. Please give a brief answer to the following questions:

1. What did you like the most about these lessons?
2. What did you dislike the most about these lessons?
3. Did you experience any difficulties while using Geogebra?

After each question there was space for students to answer as much as they wanted. Given the small sample size it was feasible for me to include open-ended questions. I used these questions in order to collect in-depth data about students’ experiences and beliefs avoiding the limitations of pre-set categories (Cohen et al., 2007). I wanted the students to decide what they wanted to report without much prompting. Thus, I had the chance to explore and generate items that could not be predicted otherwise, as open-ended questions can catch the authenticity, richness and depth of response that would not be captured with other forms of questions. Nevertheless, open-ended questions might lead to irrelevant, superficial and redundant information and require more time for the responder to complete the questionnaire, and more data handling for the researcher (Munn and Drever, 2004). However, I tried to limit these disadvantages by pilot testing the questionnaire.

**Transcribing the data**

The video-recordings transcribed were those of the 3 lessons conducted between 14th and 16th of April. These observations were transcribed according to the model proposed by Powell, Francisco and Maher (2003). This analytical model for studying the development of
mathematical thinking employs a sequence of five interacting, non-linear phases (there are seven phases but in this study only the five first phases were followed):

- viewing the video data attentively (in order to become familiar with the content of the video data);
- describing the video data (brief descriptions of the content of 3-minutes intervals);
- identifying critical events (critical events demonstrate a significant change from previous understanding in relation to the research questions);
- transcribing (produce transcripts that are close approximations to being genuine) and
- coding (a crucial part which aims at identifying themes that help to interpret data).

Additional data consisted of voice-recorded individual interviews conducted with the teacher and questionnaires completed by each student separately. The voice data from the interviews was transcribed manually into a written form while students’ responses to the questionnaires were documented more systematically. After the transcribing, the interview and questionnaire transcriptions were coded. More details are given in the next part.

Analyzing the data
The purpose of analyzing qualitative data is to determine the categories, relationships and assumptions that inform the respondents’ view of the topic studied (McCracken, 1988). The following analysis is a descriptive narrative with issues raised throughout.

The analysis (Creswell, 2003; Strauss and Corbin, 1990) began with the transcription of the interviews, observations and questionnaires. Having this (expected) respectable amount of data I tried to gain a deeper understanding of the information and refine my interpretations. After reflecting on its overall meaning, codes\(^9\) were generated through open coding. I tried to link codes to appropriate segments of the text until they no longer provided insight to the codes (‘saturate’ the codes) and thus, having less and more manageable data. Then I sorted out the

\[^9\] Codes are tags or labels for assigning units of meaning to the descriptive information collected during the research (Basit, 2003).
codes more systematically and sorted them into categories (axial coding). The purpose of open coding was to generate codes whereas the purpose of axial coding was to ‘begin the process of reassembling data that were fractured during open coding’ (Strauss and Corbin, 1990, p. 124). The process of axial coding generated a set of 14 different categories. After that I followed a grounded theory approach in order to generate theory (Glaser and Strauss, 1999). I did not want a 14-point-list but a theory that could provide in-depth answers to my research questions. The categorized codes from axial coding were used to propose core themes that reflected the purpose of the research, thus generating patterns (selective coding\textsuperscript{10}). Thus I was able to develop the codes into a theoretical model as in grounded theory\textsuperscript{11}, using narrative passage to convey the findings of the analysis.

In the analysis I also included verbatim data since I find it important to keep the flavour of the original data which is very rich in detail (Cohen \textit{et al.}, 2007). Finally, I compared my findings with findings in previous studies in ICT in order to make an interpretation or meaning of the data.

\textsuperscript{10} Selective coding is the process of ‘\textit{integrating and refining the theory}’ (Strauss and Corbin, 1990, p. 143)

\textsuperscript{11} A case study could be approached as an exercise in the generation of grounded theory although case studies are not exclusively grounded theory studies (Robson, 2002). Grounded theory is a general methodology for developing theory that is grounded in data systematically gathered and analyzed (Strauss and Corbin). Theory is emergent that predefined (Cohen \textit{et al.}, 2007); it arises from the data and data is not stretched to fit to a certain theory.
Chapter 4: Findings

As I mentioned in the previous chapter, 14 categories emerged from the process of axial coding. These were:

- BFR: break from routine
- EICS: exploratory, interactive, collaborative styles
- ESW: extenuate student’s weakness
- CSM: change in the status of mistakes
- WBA: work becomes attractive
- IE: increasing engagement
- FCA: facilitating classroom activity
- EPC: enhancing the productivity of classroom activity
- AF: adduce features
- EI: establishing ideas
- RA: raising attention
- PPA: promoting pupil autonomy
- TAR: teacher as resource
- PAR: peer as resource

Through selective coding I grouped the 14 categories into 3 broad themes, namely classroom organization and management, cognitive amplification and learning attitudes. More specific:

- **Classroom organization and management**
  Under this theme I placed categories that related to the function and operation of the classroom. As a consequence BFR, FCA, EPC and TAR were included.

- **Cognitive amplification**
  Categories that enhanced students’ learning and development of mathematical ideas were placed under this theme. These were: ESW, CSM, AF, EI and RA.

- **Learning attitudes**
In this last theme I placed categories that related to the students’ attitudes towards learning. Thus EICS, WBA, IE, PPA and PAR were included.

These three themes will now be explored in detail.
Classroom organization and management

This theme concerns the ‘orchestration’ of the classroom witnessed in the lessons and reported in the teacher’s interviews and students’ questionnaires. It associates the use of Geogebra with changes and differences in the classroom environment. These changes refer to break from routine, facilitation of classroom activity, enhancing of productivity and quality of the classroom activity and the use of the teacher as resource during students’ investigation.

Break from routine

The use of computers in mathematics lessons was seen as something of a break from routine, both by the teacher and the students. This break contributed in making the lesson more interesting, attractive and enjoyable to the students. It is linked with the category WBA; however, break from routine encompasses aspects of classroom operation thus it is placed under this theme.

The facts that the lessons were carried out in the computer lab rather than in the typical classroom and on the computer rather than on the board were catalysts for breaking from routine. Students in the questionnaires reported that:

Because they [the lessons] were on a computer rather on a piece of paper automatically made the lesson more interesting…12

I liked that we had the lessons on the computers. It is the first time that I’ve done such a thing and I really enjoyed it…

Facilitating classroom activity

Additionally, the ‘classroom organization and management’ theme associates the use of Geogebra with the facilitation of classroom activity, allowing for its components to be carried out more quickly, reliably and with greater ease. It supports the execution of tasks, produces accurate and real data, helps generate patterns more quickly and allows students to work on their own pace.

12 In this chapter ‘…’ stands for an incomplete sentence.
The majority of the students mentioned that they liked that the program helped them measure accurately and with great ease, without divergences from their classmates since the results were the same for all students:

I liked how easy it was… with just a single click we measured the diameter, the radius, the area and the circumference of the circle...

I liked that we didn’t have to use our rulers to measure stuff, which usually means that each one of us gets a different result. In these lessons we measured quickly and accurately. We cooperated with our classmates and ended up with the same results...

Loveless (1995) claims that computer software allows for exploring data since real numeric data can be carried out quickly, accurately and as many times as needed. It can present real data collected from real situations quickly whereas without technology it is reasonable, to keep the numbers simple and preferably whole. In these lessons measurements functioned as a means for finding explanations, gathering information, explaining their conjectures and drawing conclusions about the results. Measurements also provided students with specific examples that formed the ground for further conjectures and generalizations (Christou et al., 2004).

The teacher claimed that every student concluded to correct measurements and when the time came to discover relationships these were clear, convincing, and identical for all students and they were produced quickly, whereas in previous years without the use of Geogebra, students found different relationships according to their measures and that led to disagreements in the classroom:

They [the students] were secure that what they were measuring was right and every team ended up with the same conclusions. It was clear that the diameter was twice as big as the radius thus we avoided the confusions we had in previous years… [1st interview]

In previous years when I challenged the students to examine the relationship [between the circumference and the diameter] it wasn’t that accurate… they found 3.17, 3.20, 3.05, 3.07 and they had to trust teacher’s judgment that the average is 3.14 and not all of them were convinced… today it was different; they all saw that every example was 3.14… it was clear and convincing… [2nd interview]
This **looking for and finding patterns** is fundamental to the development of an understanding of mathematics. Geogebra offered ways of providing speedy access to pattern generation which allowed students more time to concentrate on the patterns themselves (Briggs and Pritchard, 2002). Geogebra offered students opportunities not only to discover patterns but also to generate them quickly. The following extract comes from the second lesson observation and gives the conversation between two students (a team) sitting next to the camera that videotaped the whole classroom:

Achilleas: In the first three we find the same number. It’s 3.14.
Paris: I think that all of them would have the same number, 3.14.
Achilleas: You think? Let’s try the rest and see.
Paris: The next one is 3.14 as well. That’s it, I’m sure, it’s a pattern. We discovered a pattern.

The teacher also mentioned that the program facilitated the **execution of tasks** which aided students’ work, teacher’s job and the lessons’ goals, leading to a more scientific lesson:

In the traditional classroom you cannot do so many things… in these lessons students worked and in little time produced results… I believe this program facilitates a lot teacher’s work… Comparing this with previous years that I taught these lessons I believe that it facilitated the students… the program is friendly both for the student and the lesson’s goals… This way of carrying out the lessons is more scientific, more mathematical… indeed I could see more professionalism than in the typical classroom… [4th interview]

What also facilitated the classroom activity was that the use of Geogebra allowed **students to work at their own pace**, without fearing that they are holding back or having to wait or work slower to keep up with the others. In the normal Cypriot classroom it is very possible to have a mismatch between the learning style of a number of the students and the learning taught by the teacher. The teacher reported that:

Some students delayed while others proceeded, so I had to adapt a medium speed that didn’t facilitate either. In contrast, today everyone worked at their pace… I did not see a student holding totally back. I did not see even one team that didn’t finish their tasks; they all managed to finish, some of them faster and some later… [1st interview]
Every child learns in their own way and at their own speed, thus in the Cypriot mixed-ability classrooms the teacher needs as much help as possible to create a learning environment that provides access to mathematics for all the pupils, whatever their learning style may be. Geogebra can serve as a tool that can give the teacher the means of providing a learning environment for all learning styles (Way and Beardon, 2003) through exploration, collaboration, guidance, questioning and explanation. Computer work allows the students to work at their own pace, according to their own needs and their own abilities (Mcdonald and Ingvarson, 1997). DfES (2002a) argues that the use of ICT facilitates independent learning, while the ability to work at their own pace and their own time is ‘appreciated by students for its flexibility and the opportunity to revisit course material, to reinforce classroom learning or for purposes of revision’.

**Enhancing productivity of classroom activity**

The theme ‘classroom organization and management’ also associates the use of Geogebra with *enhancing* the pace, **productivity and quality of classroom activity**, which saves time and increases the number of examples in a single lesson. The teacher reported that:

> Today students had the chance to investigate 10 different circles and not only 2-maximum-3 circular objects as in previous years when you had difficulties convincing them that is true...this way was more mathematical, it was a proof for the students… We didn’t lose time arguing about the centimeters and the way students worked was more reliable... we gained time… [2nd interview]

In addition she noted that:

> In less time you can do more things… it helps you gain valuable time… [4th interview]

Oldknow and Taylor (2000) claim that ICT use in the classroom allows more time for student-work and generally improves school efficiency while the introduction of computers in the classroom increases the amount of things students do and learn in a specific time period (Cordova and Lepper, 1996).
Teacher as resource

This theme also associates the use of Geogebra with teacher’s role. **Teacher as resource** involves students seeking information from the teacher and receiving prompts and guidance from her. Despite the fact that technology allows for more student-centered approaches, the teacher’s role remains absolutely central in reflecting the experiences and work of pupils (Pachler, 2001).

The teacher adopted a variety of roles in the classroom: she provided initial information to support and encourage pupils in their new techniques and ideas. She then took an intentional non-interventionist stance to promote exploration and autonomy, and then returned to make suggestions to broaden students’ learning. Students worked collaboratively on the computers during the main part of the lesson. Even though the students explored mathematics in groups the teacher was still an advisor in the background and gave support when help was needed. She did not stand back and let it happen, but used a variety of questions and prompts, giving and withholding information at different times to support and increase pupil autonomy. She found a balance between allowing students the freedom to work on their own and dealing with any misconceptions. This scaffolding from the teacher not only helps students to accomplish the task successfully, but also places the student in a situation which leads them to eliminate their need for help (Hoyles, 1991). Due to this fact and students’ familiarization with the software, from the observation it is evident that they reduced the use of the teacher as resource and made more use of their peers as resource. However, this category encloses aspects of collaborative learning behaviour thus it is placed in the theme ‘attitudes towards learning’.

The introduction of computers into mathematics classrooms modifies pupils-teacher relations with pupils more likely to make decisions (Hoyles and Sutherland, 1989). The presence of Geogebra did not deprive the teacher of responsibilities but presented new ways for thinking about the teaching process. She played a vital role in providing appropriate structure (without that implying authoritarianism), direction and guidance in pupils’ learning. She was a mediator clarifying the mathematics in Geogebra and the extension of the ideas encountered (Clements and Battista, 1989). Without this intervention by the teacher many misconceptions can persist (Hoyles and Sutherland, 1989). Papert (1980) argues that this computer environment offers opportunities for more articulate, effective and honest teaching relationships.
The last two components of this theme concern practical issues on the use of Geogebra. Both the students and the teacher reported the need to familiarize themselves with the software in order to use it more confidently. Students reported that:

Sometimes I got lost; I had difficulties finding the right tool. Gradually, I got familiar with the software.

At the beginning, I found difficulties using the software because I wasn’t used to it.

Moreover, the expansion of the use of Geogebra was apparent in the observations. Students while solving their exercises, a simulation of which was provided in Geogebra, expanded its use. Instead of applying the formula manually they used the measuring tools concluding to wrong answers. That means that to use Geogebra it is necessary for the students to know how it represents, manages or computes information (Balacheff, 1993). It is insufficient just to give students tools to carry out procedures if they are not able to make sense of the relationship between the various processes, concepts and representations (Tall, 1998).

The teacher reported the need for the students to familiarize themselves with the software, learn how it works, and understand its logic and how the tools can be used. She also reported that the exercises prepared in this software must be given a lot of attention before implementation and that if she was going to keep using the software in the classroom she needed support:

Some students found difficulties in drawing the diameter, moving the quarters of the circle and creating a new one but again it was a matter of familiarization with the software. They needed some time to become familiar… [2nd interview]

I didn’t have any difficulties incorporating the software in my teaching… It can be implemented very easily. The way I saw it working in the classroom, I believe that it can be easily implemented and used in the classroom… It’s just that as a teacher I need advice and training to get to know the software and organize my lessons as we did in these three lessons… If it is going to be used constantly in the classroom, students must have training lessons about the symbols, the windows; definitely training lessons for the students in order to become familiar… Firstly students need to become familiar with the software and secondly we have to be very careful about the exercises we design on the software. We have to consider the exercises a lot before using them in the classroom in order for the students to construct real knowledge. Thus
on the one hand we have to be very careful and specific about the content and on the other hand
students have to be familiarized with the software… [4th interview]

When it comes to integrating Geogebra teachers should make sure that it teaches what students
are supposed to learn, enhances the learning experience, is functional and operational for the
students, appropriate to the students’ age, has accurate content and matches the ability level and
language level of students (Williams and Easingwood, 2004). For the majority of teachers,
providing technology is insufficient; teachers need training and support to integrate
technology into their teaching and to develop successful technology-assisted teaching practices
(Cuban, Kilpatrick, and Peck, 2001). Teacher education and professional development about new
technologies is needed in order to prepare teachers for their use and demonstrate them how to
effectively integrate them into their everyday teaching (Preiner, 2008).
Cognitive amplification

The theme named ‘cognitive amplification’ concerns the reinforcement and enhancement of students’ learning. It associates the use of Geogebra with the ability to provide cognitive power tools that improve students’ construction of knowledge opening up new possibilities of thought and action (Pea, 1985). In this work these tools are used to alleviate students’ weaknesses, change the status of mistakes, provide results, adduce features, establish new ideas and raise attention.

Extenuate students’ weakness

Cognitive amplification associates the use of Geogebra to extenuate students’ weaknesses. The teacher reported that the use of Geogebra improved opportunities for students with less developed kinetic abilities and helped lower-achievers to participate in the lessons:

Some students, who find difficulties understanding concepts theoretically or by seeing them on the board, suddenly became active and worked on the computers. Secondly, some students who didn’t have the kinetic skills required to measure accurately were delayed in previous lessons. But now they were secure that their measurements were correct… [1st interview]

Today everybody worked; there wasn’t anyone who wasn’t working whereas usually only the students with the better developed skills participated… [2nd lesson]

Students who in previous years couldn’t realize what we were doing they were able to solve them [the exercises] today … they found it very easy to solve them and managed to, what we say, ‘I understand what I’m doing and can I transfer my knowledge’… [2nd lesson]

In the classroom students cooperate but when it comes to measuring and drawing their skills are not the same. One would be working and the other would just observe. In contrast, in the lessons conducted, the cooperation was equal… In a traditional lesson you will challenge the students and they will work in teams but again the low-achievers will follow the high-achievers. Even in a group of four you see that one or two students support its function. The rest of the students listen, observe; the lesson is more pathetic… [4th interview]

It seems that the use of Geogebra in mathematics lessons transformed some students from inactive observers to energetic members of the classroom. With the use of Geogebra students are
not passive recipients but active participants; they can make decisions, see consequences and receive feedback (Pachler, 2001). Many researchers have shown ways in which ICT can support pupils with learning difficulties, physical disabilities, additional or special educational needs (BECTa, 2007b).

*Change in the status of mistakes*

This theme also associates the use of Geogebra with a **change in the status of mistakes**. The use of Geogebra facilitates the students’ corrections and also gives them the opportunity to remove evidence of their mistakes which might attract unwelcome reactions from the teacher or their classmates. The conversation below shows this easy access to correction:

Aris: (he goes to the third group of tools and picks the tool ‘line between two points’.
   Instead of drawing a segment-radius he draws a line).
Ermis: Give it to me (the mouse). What have you done there?
Aris: Leave it there. We will correct it. Press undo.
Ermis: Which tool do we want?
Aris: The segment between two points
(They are searching to find the tool)
Aris: There it is.
Ermis: (he picks the tool and draws a radius). Like that?
Aris: Yes like that. It’s easy.

While working on Geogebra trying to discover relationships, students knew that they could express their ideas on the dynamic sheet and get ‘neutral responses’ from the software (Holzl, 2001). Geogebra can favour the implementation of the pupil’s intuitions (Hohenwarter and Fuchs, 2004). Students can trust their intuitions, test them and change anything if something seems to go wrong and eventually come to a solution. They feel that they can try out everything that seems logical to them without fear of making a mistake. Young children can gain many benefits from implementing their intuitions, trying tools and seeing them put into practice. This is something that is enormously satisfying to children, and at the same time encourages the use of logical thinking and engagement with mathematical activity (Briggs and Pritchard, 2002).
The change of status of mistakes relates with the ability of the computer to provide feedback to the students. There is evidence to show that feedback from a computer can improve pupils’ learning (Higgins, 2001b). The learners can have **immediate and dynamic feedback** that provides the opportunity to try different possibilities, see the consequences of their decisions, correct their mistakes and plan their next moves accordingly and this is fundamental to the learning process (Atkins, 2003; Loveless, 2003). Geogebra provides visual and conceptual feedback to the learner; feedback results from the disagreement between the anticipated figure and the produced figure. This form of formative feedback evaluates students’ actions without criticizing them (Hoyles, 1991) and it is essential in supporting pupils to identify how they can improve and correct their errors.

This change of the status of mistakes underlines the **provisionality of results**, something that exists in many software programs and provides an environment open to revision (Ruthven and Henessy, 2002). Through this provisionality, students have the opportunity to modify, try out alternatives and keep track of their actions and ideas (Allen, 2007).

*Adduce features*

The use of Geogebra to amplify cognition is also associated with the promotion of features. **Features adduced** refer to visual representations that develop visual imagery, dynamic images and effects through which features of mathematics adduce helping the communication of meanings. During the lessons students could enlarge or shrink shapes observing how relationships are formed, construct shapes and reconstruct them in their components, reflect on mathematical relationships and discover patterns, thus constructing mathematical knowledge. Students mentioned that:

…It was better seeing the circle transforming and observing relationships rather than seeing a still circle on a piece of paper….

[I liked it] when we discovered the area of the circle in the last lesson. We moved the runner and the circle opened, flipped and we discovered the area of the circle.

…I also liked it when we were changing the circle and observed the relationships; it was better than seeing a still circle on a piece of paper and having to guess at the relationship.
Additionally the teacher reported:

…The examples, the exercises were remarkable. At this stage the software was really valuable. Having the possibility to move the quarters and construct a new circle; this is something that not many students can imagine. It requires high levels of abstraction that not all students can reach. Most of them couldn’t understand it but today they could take the pieces, bring them together and construct a new circle… [2nd interview]

…Assembling shapes, the fact that students see a shape from different perspectives, they can see it reconstructing and constructing in front of their own eyes, enlarging, shrinking while the relationship remains the same. The theoretical part becomes visual, they visualize and that is very beneficially… It [Geogebra] has the potential to present the same thing in different ways and generally it has incredible potential, constructing, reconstructing, expanding, shrinking, flipping. A right-angle triangle for example. All students draw it with the right angle on the base of the triangle; they think that if the right angle is not on the base then it is not a right-angle triangle. Using this software you have the chance to flip the triangle and students can see that the same triangle will move up, down, left, right. You are not lying any more, they can see it with their own eyes and this visualization is extremely important it primary school… [4th interview]

Visualization here refers to visual perception and by extension to visual imagery; it involves two essential cognitive functions: epistemological function, nothing is more convincing than what is seen, and synoptic function, it seems to give immediately a complete comprehension of any object or situation (Duval, 1999). Visualization is an essential part of mathematics learning and Geogebra can play a powerful role not only in stimulating and shaping students’ visual images but in providing access to new forms of representation as well as to multiple and linked representations (Kaput, 1986).

There is clear evidence that geometrical ideas are difficult for many children; many misconceptions appear to be related to a failure to view concepts in a dynamic rather than in a static form (Noss, 1987). Geogebra can present information in different ways by showing it in different forms or by enabling changes to be shown dynamically (Ainsworth et al., 1997) widening the range of possible activities and providing deeper reflection, exploration and
heuristics than in a paper and pencil approach (Straesser, 2001). This dynamic environment enables students to explore the problem and make mathematical conjectures.

Geogebra provides an active way to manipulate visual mathematical objects allowing understanding of concepts (Tall, 1998). Dragging makes these environments much more powerful than traditional paper and pencil learning; drag capability allows students to experience the direct manipulation of geometrical objects created on the screen (Marrades and Gutierrez, 2000). Students have the opportunity to see dynamic representations of geometric configurations that cannot be easily illustrated without the use of technology. Dragging a geometrical object enables students to check its characteristics and attributes (Hadas et al., 2000). Observing changes can develop pupils’ understanding of mathematical relationships and convince the student of the truth of the conjectured attribute, thus developing conceptual understanding (Jonassen, 2000). Children who see shapes build them and carry out transformations of them on a computer screen, take these images and construct their own knowledge of shapes developing their understanding of mathematical relationships (Fey, 1989). Enabling children to see something happen dynamically can support the formation of mental images which will in turn assist in the process of understanding (Briggs and Pritchard, 2002).

Establishing ideas

Cognitive amplification also associates the use of Geogebra with the establishment of ideas. This refers to the formation and consolidation of ideas by the students while justifying their decisions and conjectures. Students reinforce their learning and establish mathematical ideas while working on the computer, exploring and discovering relationships, discussing and reasoning about their results and participating in whole-class discussion. This category is linked with pattern generation in facilitating the classroom activity but focuses more on the identification of ideas from the patterns generated rather than on the access to pattern generation. The following quote comes from a conversation between Aris and Ermis during the second lesson:

Aris: Make the first division. 25.13 divided by 8.
Ermis: 3.14125. Should we write all decimals?
Aris: Write the first two. I think we will be ok.
Ermis: 3.14 then.
Aris: Go to the next one. 43.98 divided by 14.
Ermis: 3.1414... It’s the same
Aris: You are right. It’s same… You’ll see… everything would be 3.14.
(They try 3 more circles and decide that this is true for all circles. They complete the table)
Ermis: So the length of the circumference is equal to…?
Aris: Wait... The length of the circumference divided by the diameter is always 3.14.
Ermis: Let’s think…
Aris: Equals diameter divided by…
Ermis: It’s not division it’s multiplication
(While they are thinking Paris comes, they discuss their discovery of 3.14 and then he leaves. The two students continue thinking about what the circumference of the circle is equal to. They look at the table.)
Ermis: The length of the circumference equals diameter multiple by…
Ermis: You are right. That is because every time we divided we always found 3.14.

Concerning the establishment of ideas the teacher reported:

I believe that the students understood it… they didn’t gain a shallow understanding but they proved the relationship to themselves. By measuring and observing the results they understood the relationship and then they applied it in the exercises. I believe that they had an in-depth understanding of the lesson… [2nd interview]

Students saw that in every example they found 3.14. They concluded that since this number is constant and is not changing, the circumference of every circle is 3.14 times bigger than the diameter and they said that they don’t have to measure the circumference, the only thing they need is the diameter multiplied by 3.14… I believe that they will keep these lessons longer in their memory than a traditional one… [2nd interview]

I believe that I achieved higher levels of understanding, students achieved a more in-depth understanding and can transfer this knowledge to other circumstances; to look for relationships and to search how formulas are discovered in mathematics… students constructed knowledge and established ideas on preexisting knowledge… [4th interview]
GeoGebra was created to help students gain a better understanding of mathematics since it can foster mathematical experiments and discoveries in classrooms (Hohenwarter and Preiner, 2007). Since a lot of emphasis is given in ‘understanding’:

*Understanding means having a great deal of prerequisite knowledge at one’s fingertips; it means having multiple perspectives on the objects involved.; it means having multiple representations for them, and coordinated means of moving among perspectives and representations. And it means having all this knowledge organized in ways that derive power from redundancy* (Schoenfeld, 1990, p. 4).

Thus understanding is a complex process in which didactic, cognitive, epistemological and mathematical aspects intertwine in determining the processes leading to understanding (Dreyfus, 1993). The dynamic environment fostered the interaction between construction and proof, between doing on the computer and justifying the conjectures with mathematical arguments making students more systematic in their conjectures (Laborde, 2000). It provided a bridge to understanding through exploration leading to the acceptance of the idea by providing a dynamic representation of the mathematics involved (Hoyles and Sutherland, 1989). Evidence from researchers shows that ICT can support mathematical learning and teaching, encourage mathematical thinking, enhance children’s learning (Smith, 1999), raise pupil attainment (Moseley *et al.*, 1992) while the knowledge remains long-term in students’ minds (Holzl, 2001).

**Raising attention**

Lastly, under the theme ‘cognitive amplifier’ stands the association of the use of Geogebra with the stimulation of attention. **Attention raised** refers to the use of Geogebra to focus students’ attention on overarching ideas and processes and free students from producing the data avoiding subordinate tasks. The use of Geogebra in these lessons helped students in their measurements since they were automatically produced by the software with the use of certain tools, thus allowing more time for the students to focus on finding relationships and discovering patterns. The teacher reported:

> If we didn’t use this software, in a traditional lesson, students counted squares in given circles and they approximately ended up to the area of the circles but that was a very time-consuming
procedure that changed the aim of the lesson. Instead of wondering about how to find an easier way to calculate the area, they were wondering about how many squares are in the circles. Our goals changed and when reaching the point with the radius, the square of the radius and its relationship with the area we were losing the point of the whole thing. In contrast, today the lesson was more focused to the aim. The convenience offered by the software to calculate accurately and without any doubt the area of the circle, and to try out many circles, and then to wonder about where this result comes from; I think that helped them a lot. We avoided the time-consuming and pointless procedure that led us away from our aims and we were more focused on our goals… [3rd interview]

In previous years we had to find the average of our results from the measurements. And as I noted before the lesson got away from its goals. The lesson was transferred to what is the average, how do we find it and got tangled in conversations that did not facilitate the lesson’s goals. In contrast, today we avoided all these; we had this convenience; to easily find the area without any interventions… [3rd interview]

In addition, students stated that:

It measured the circumference, the area, the radius and the diameter automatically and that helped me understand more things about the circle.

Geogebra provided an environment in which the learner physically experienced the ideas of the mathematics at a fundamental level without the need to concentrate on the symbolism and the computations required to produce a solution (Tall, 1998). Hohenwarter and Fuchs (2004) argue that this gives teachers more time during the lesson to concentrate on fundamental ideas and mathematical reasoning. Geogebra offers automatic functions that provide the opportunity to the student to calculate, measure, draw etc with great ease. These automatic functions of Geogebra allow storing, changing and displaying information, analyzing and synthesizing information at higher levels leaving students time to think about the information presented (Loveless and Dore, 2002). Students are freed to think about the implications of the information with which their dealing and not lose valuable time producing the data (Fey, 1989). This process of abstraction includes the isolation of properties and relationships and requires the ability to shift attentions from the objects themselves to their properties and relationships (Dreyfus, 1993). Thus students
are more focused on the mathematical aspect on the activity rather on the practical; I am not stating that this is always beneficiary but it was clearly useful in these lessons.

In conclusion, Geogebra permits activities that need high level thinking; it entails pupils engaging with potentials such as alleviation of students’ weaknesses, change of status of mistakes, provisionality of results, promotion of mathematical features, establishment of ideas, a rise of attention, users learning from feedback, making connections and working with dynamic images.
Learning attitudes

This theme concerns students’ attitudes towards learning and their learning behaviour. These learning attitudes relate to aspects such as students’ self-confidence, self-esteem and independence (OECD, 2004). It associates the use of Geogebra with the ability to provide exploratory, interactive and collaborative styles, consider peers as resource, make classroom work an enjoyable experience and increase students’ autonomy and engagement in classroom work.

Exploratory, interactive and collaborative styles

The theme ‘learning attitudes’ associates the use of Geogebra with the provision of exploratory, interactive and collaborative styles. During the lessons students engaged in activities in which they investigated, interacted, discovered and cooperated with their peers. It is these styles that led the students to characterize the lessons as fun, easy, exciting, not boring and gave them the sense of playing around:

I liked it when we observed relationships between the radius, the diameter, the circumference and the area…

I liked cooperating with my classmates…

The teacher noted that:

Students wondered about the circle circumference. They experimented and discovered the relationship between the diameter and the circumference and began to understand the concept of ‘π’… [2nd interview]

Students wondered about the area of the circle, how we can measure it and saw that it was difficult to find it. Then working on the software, they observed the relationship of the square of the radius with the area and discovered ‘π’… They understood the difficulty in measuring the area and the thinking procedure, how mathematicians discovered this formula. They followed the same exploratory path as mathematicians… [3rd interview]

The software facilitated group work and cooperation and these assisted my teaching… students understood the process of looking for patters and search for how formulas are discovered in
mathematics… Students followed the same exploratory path as a scientist; observing, setting the
problem and searching for its solution by exploration with the help of the software… [4th
interview]

One of the most welcome facilities of dynamic environments is their potential to encourage
students’ research. Polya (1945) emphasized the connection between these explorations with
deductive reasoning. In this exploratory approach, students are inducted into the acquisition of
mathematical ideas and deductive proof (Christou et al., 2004). Students could experiment
through different dynamic tools the geometrical objects they constructed and were able to infer
properties, relationships and formulas.

Moreover, there was greater collaboration and task-related interaction when students worked
with the software (Wegerif, 1998). During the lessons students demonstrated collaborative
behaviour and had the opportunity to develop their skills of negotiation, observation and
interpretation as well as social skills such as sharing ideas. Each participant of every group
brought their own experience and background to the situation and talked about their ideas and
considered what their peers said in order to arrive at a group work solution. This environment
stimulates collaborative learning, promotes richer and deeper interactions than are commonly
seen in traditional lessons enriching and facilitating the interaction between all participants
(Papert, 1980).

This is linked with the previous theme, cognitive amplifier. These social interactions can play an
essential role in facilitating cognitive growth. Cognitive change may result from situations that
occur while students work collaboratively. When individuals engage in a process of ‘reciprocal
sense-making’ (Nastasi and Clements, 1992) they attempt to extract meaning, generate ideas, or
solve problems through discussion and effective learning takes place through this social
interaction and interpersonal support (Pachler, 2001). Vygotsky supports the idea that individual
cognitive development results from social interaction in the world and that speech, social
interaction and cooperative activity are all important aspects of this social world (Vygotsky,
1962).

Geogebra can help students to foster experimental, problem-oriented and discovery learning of
mathematics (Hohenwarter and Preiner, 2007). The computer as used in Geogebra is interactive
and this interaction is social since it promotes cooperative student work and student interaction with their peers. Many fear that the use of technology in classroom will cause social isolation or reduction of students’ social skills. However, researchers showed that ICT promotes students’ interaction, collaboration and discussion (Agalianos, 2001; Light and Blaye, 1989). There is also evidence that computers can support pupils’ speech and improve their discussion when they collaborate (Wegerif and Scrimshaw, 1997).

Geogebra also provides opportunities for investigations. This contributes to the acceleration of child development since students are controlling rather than passively observing the new knowledge (Holzl, 2001). Moreover, our fast-changing technological society requires individuals who learn through experience and investigation; people who have a positive approach to problem-solving, flexibility and transferability in new situations (Loveless, 1995).

Work becomes attractive

The theme also associates the use of Geogebra to prevent work from becoming unpleasant. The use of Geogebra contributed in making the work become attractive and enjoyable and in raising interest and stimulation. The lessons’ observations show that students were well stimulated and as the teacher reported their concentration was noticeably better than usual:

The fact that students used computers automatically made the lesson more interesting… We gained interest… It was interesting and students left excited and later when I talked to them they told me that the lesson was very interesting to them… [1st interview]

Students have already indicated that they are very satisfied. Today we needed more time to finish the lesson, the bell rang and no one stood up to leave, something which is very rare… [2nd interview]

Many students noted that the lessons were more interesting and enjoyable:

I liked it when we discovered the area of the circle and generally the software was something that excited us all...

It is easier and more interesting this way than drawing and measuring on a piece of paper on my own…
The lesson was easier and more fun…

It was interesting having the lessons on the computers. For the first time mathematics weren’t boring...

I liked everything because it is a new way of learning…

The use of Geogebra made the lessons easier, more appealing, fun and fascinating for the students. English (2006) argues that ‘**ICT enables you to access or create stimulating resources, to distribute or display them attractively to individuals, groups and whole classes of pupils, to capture the interest of your pupils and motivate them to want to learn and to carry out many of your administrative tasks more effectively and efficiently**’ (p. 9).

**Increasing engagement**

Learning attitudes associates the use of Geogebra with the **increase of engagement** in classroom work. The lessons’ observations showed that students engaged deeply and strongly in classroom activities. The teacher reported that:

Students were active all the time, all of them were working throughout the investigation… they were active the whole time and when the bell rang they didn’t leave, something which is unfamiliar… [1st interview]

There is evidence that the use of technology in the classroom impacts positively on pupils’ attitudes towards their engagement with their school work and generally their engagement in learning. Students become more deeply involved in the activities showing higher subsequent levels of aspiration (Cordova and Lepper, 1996; DfES, 2003). Moreover, evidence shows that ICT can have a positive impact on students’ levels of concentration, helping to reinforce learning (English, 2006).

**Promoting pupil autonomy**

The theme learning attitudes also associates the use of Geogebra with the **promotion of pupils’ autonomy** and the use of their peers as resource. Students were more confident to assert control of their own learning without the constant need of the teacher. Students sought the teacher’s help especially in the first lesson but after students were familiar with the software they were more
autonomous. In the second and third lesson where new tools were introduced students did not ask the teacher as in the first lesson but searched to find them on their own and they continued the activities without any interruptions. On the videotapes one can see Aris and Ermis (the videotaped students) discussing, exchanging thoughts and drawing their own conclusions on relationships, formulas and problem solving. In the questionnaires students report that they liked the fact that they worked and explored on their own:

I liked it that we measured the circumference, the radius, the diameter and the area of the circle discovering relationships on our own…

The teacher reported that she saw the students asserting control and working autonomously; after challenging them they investigated, drew conclusions and solved the exercises on their own, and proceeded through the activities promptly without the need of her constant attention:

In a traditional lesson in the classroom you will choose the basic stuff, you will use some objects and the board, try to prove it [the formula] mathematically on the board and all students will have to follow the teacher… In these lessons students explored and the formula. I didn’t say to the students: that’s the formula, learn it, see that, multiply that by that and learn it. Students discovered the formula on their own… [4th interview]

Teaching with Geogebra hands over a high degree of autonomy to pupils who work on their own and at their own pace (Agalianos et al., 2001) and as Papert (1994) claims the best learning takes place when the learner takes charge. Students working on computers have more control of what they do because they can try things out as opposed to having the teacher as the only source of information (Agalianos et al., 2001).

What is important in the software implementation is establishing a teacher-pupil relationship in which the pupils learn to assert control of the computing activity and do not turn to the teacher for the right answer (Hoyle and Sutherland, 1989). Evidence shows that the use of technology in the classroom can have a positive impact on students’ independence and autonomy helping them reinforce learning (English, 2006; Hoyle and Sutherland, 1989); it can provide an environment in which the learner is in control and in position to investigate and control their learning (Loveless, 1995). Moreover, students’ autonomy over their learning fosters social interaction and increases their self-esteem and confidence (Kull, 1986).
Peers as resource

Finally, this theme associates the use of Geogebra with students seeking help from their peers. Peers as resource is connected with the previous category ‘pupil autonomy’ since in both cases students are more in control of their learning. However this category focuses on the process of seeking information from one’s partner rather than acquiring autonomy.

Students often used their teacher as resource but as they got more familiar with the software they reduced the use of this source and used their peers as resource. Students in groups spent a short time getting familiar with the environment, asking for help from the teacher and then started to use strategies that became more sophisticated and sought help from their peers.

Seeking information from one’s peers accounts for higher-order thinking (Nastasi and Clements, 1992). Research in cognitive psychology has shown that learning occurs when the learner is engaged in some sort of cognitive elaboration (Wittrock, 1986) and one of the most effective means of elaboration is explaining the material to someone else which is what happened when the student provided information to their peers. Evidence shows that students are more likely to interact with peers, engaging in group problem solving and sharing when using computer software (Fire Dog, 1985). Vygotsky (1978) refers to the intellectual value of peer support arguing that a partner can achieve a level of potential development rather than a level of actual development.
Chapter 5: Discussion

The aim of this study was to investigate the potential and the implications of the implementation of Geogebra for teaching primary mathematics. Within this framework I set out to explore how the classroom dynamics are shaped, the potential of Geogebra for mathematics teaching and learning, students’ learning attitudes and the feasibility of its use in the primary classroom. From the analysis of classroom observations, teacher’s interviews and students’ questionnaires I found that the introduction of Geogebra influences the educational practice in three dimensions, namely, classroom practice, cognitive development and learning attitudes. This theory did not precede the research but was generated from the data gathered.

The findings from this study demonstrate that classroom management breaks from routine while using Geogebra. It facilitates the classroom activity and enhances its productivity and quality. Geogebra can be used to facilitate the display of information, to increase access to pattern generation, to assist the execution of tasks and to provide students the opportunity to work at their own pace. At the same time the teacher’s role remains central and essential in the classroom practice. The teacher’s role is based on identifying appropriate learning outcomes, choosing appropriate software activities, structuring and sequencing the learning process. Issues concerning training and support for teachers and students also emerge. Providing Geogebra and technology in general to schools or teachers will not necessarily make a difference. The way that it is used by pupils and teachers is what makes the difference. Computers should not be used so that one can marvel at their brilliance; they should be used to help develop a better mathematical understanding. Thus, professional development for in-service teachers and training courses for students needs to be adopted. Teachers need to be informed how to effectively integrate Geogebra into mathematics teaching and students need to gain knowledge of how the software represents and manages information and generally how it works.

It appears that while there are certainly no guaranteed results, Geogebra has the potential to improve students' educational experiences; it can enable the effective application of constructive, cognitive and collaborative models of learning. Geogebra is not just a mathematical tool but also a tool for thinking helping to enhance students’ learning. It can serve as a vehicle for helping students to foster fundamental geometrical concepts. These lessons show that the use of
Geogebra in an open setting has a mediating role for students’ reinforcement of mathematical ideas and construction of knowledge. It contributes to students’ use of their mathematical knowledge and stimulates them into making their thinking visible. Their attention is more focused on overarching ideas; they do not waste time on secondary tasks while their weaknesses are alleviated. The nature of mistakes is also changed and opportunities are provided to students through provisionality of results and feedback to correct them and move forward. Students receive neutral responses and they are not worried about the consequences of making mistakes. Mathematics with Geogebra seems to have a new, more flexible structure. It widens the range of possible activities, provides an access route to deeper reflection and exploration than in paper and pencil practice. Understanding of mathematical concepts and relationships can also be achieved through the visual manipulation of objects provided by Geogebra. Although obstacles to understanding mathematical concepts do not disappear, the achievement of the students in these lessons should not be underestimated.

The findings of the research also reveal that the use of Geogebra can provide rich mathematical environments in which students are engaged in classroom activity. It appears to have the potential to facilitate peer interaction, as well as to focus that interaction on learning. Elementary students engage in collaborative activity during the use of Geogebra, and this collaboration is equal between peers. Their autonomy seems to be increased. They see themselves as learners, develop confidence to try things out in an experimental manner and are motivated to seek justifications for their conjectures (Gardiner et al., 2000). Geogebra can motivate students; it is motivational because it enables pupils to make improvements to the quality of their work and has a positive effect on students’ behaviour because its use makes pupils more committed to the task and allows them to feel more in control (DfES, 2003). Students are more deeply involved in the activities which become more attractive and enjoyable, and thereby learn more from the activities in a fixed period of time. Generally, pupils responded positively to the use of Geogebra: they engaged well with lessons, their behaviour was good and their attitudes to learning were very good.

This case study is an honest account of a real lesson with ordinary students. I wanted to avoid showing an idealized video of classroom activity in which a perfect teacher teaches a class in
which all students are high achievers. The classroom consisted of students with mixed abilities and different learning styles. Students worked collaboratively throughout the activities. After my experience with Geogebra, I believe that although such results are not consistent and generalizable, the effort needed to incorporate it in the classroom is worthwhile. Such software, which encourages teachers to examine the learning process, allows students to assume personal responsibility and provides options in flexible and open-ended environments, cannot be easily rejected. Geogebra is not a panacea but I personally believe that it can be a catalyst in mathematics learning and teaching in Cypriot classrooms.

**Limitations**

The basic restriction of the research is that it is a single instrumental case study performed by one researcher over a small period of time. The study occurred in a particular time and under particular circumstances. The results could have been more reliable and contingent across the Cypriot student-population if more than one researcher participated, if Geogebra was implemented for a longer period of time and in different age groups and if more than one case was selected. Although these findings cannot be generalized to the overall population, because of the small number of research units, according to Yin (2003) they can generalize theoretical propositions and this was the aim of this study.

Moreover, although both the students and the teacher had training courses on Geogebra, more time was needed to have adequate training and familiarization with the software. Although I helped the teacher to effectively integrate Geogebra into mathematics teaching, she needed more time to get to know how the software works. The same is true for the students: even though I provided help in order for them to get to know Geogebra, they needed more time to understand the basic concepts on which Geogebra operates.

Additionally, the major issue here is that great caution and self-awareness must be exercised by the researcher while conducting qualitative data analysis for the analysis and the findings not to say more about the researcher than about the data (Cohen *et al.*, 2007). Throughout the research and especially during the analysis I tried to be as objective as possible in order to present a real account of the research and not an evaluation of the effectiveness of the intervention and finally conclude to un-biased findings.
This research has offered an insight into the ways a learning environment in a Cypriot primary classroom is shaped with the implementation of Geogebra. However, there is a great caution concerning the distribution of the sources used. Copying the sources and using them in a classroom cannot guarantee the same results. Factors like competence in using Geogebra and the adoption of a constructivist approach to learning (in part of the researcher and the teacher) played a vital role in concluding to these results. With these sources I do not intend to offer a ready formula but a potential for the use of Geogebra for teaching mathematics in primary school.

**Recommendations for further research**

The results of this study show that there is a good potential in using Geogebra in teaching primary mathematics in Cyprus but further research investigating mathematical ideas developed by students through the use of Geogebra is necessary. Moreover, research on teachers’ professional development in order to use Geogebra effectively in the classroom is needed. If Geogebra is going to become an established part of the curriculum it is important to continue to address the following issues: the general impact of Geogebra activity on mathematics learning, the variety of pupil approaches to Geogebra, the implications of discussion and collaboration between pupils while using Geogebra, gender differences while using Geogebra, the use and developing understanding of mathematical ideas and the role of the teacher in Geogebra environments. Finally, longitudinal studies need to be carried out to examine the long-term effects of the use of Geogebra in students’ mathematical attainment and achievement.
**Epilogue**

The world is becoming a much smaller place characterized by globalization and internationalization of the economy and society (Pachler, 2001). Globalised knowledge permeated by ICT means that life patterns are changing. The changes made by these processes to social practices are reflected in education. The demands of an emerging information society saturated by ICT challenges the future role of education. Thus, education needs to provide young people with the ability to learn how to learn, to experiment and collaborate providing the opportunity to meet the challenging requirements of the information society. Education also needs to create new types of literacy to enable pupils to survive in this increasingly demanding world. Our responsibility as educators is to equip pupils with the skills needed to live in an information society (Younie, 2001) and the use of ICT in the classroom seems to fulfill these demands:

*New ways of working have been introduced and, in a world where time is often literally money, the emphasis on getting a higher and quicker return from the capital invested is paramount. If technology can do that for industry, why can’t it do the same for education? The answer is, of course, that it can. It just takes the same leadership, drive and enthusiasm for change and the will to harness the opportunities that technology presents. By using the latest ICT equipment in our schools, colleges and universities we can be sure that technology will hold no fears for the workers of tomorrow. The challenge now, working alongside our partners, is to create a world where every child, young person and adult has the chance to learn through modern technology however and whenever is appropriate – in schools, at home, at college or in the workplace (BECTa, 2007a, p. 3).*
References


(Eds.), Professional Development in Education: new paradigms and practices (pp. 35-65). New York: Teachers College Press.


NCET. (1997). *Implementing IT*, NCET.


TTA. (1999). *Using ICT to meet teaching objectives in art - ITT primary*. TTA
http://www.canteach.gov.uk/community/ict/exemplification/index.htm


Internet

The Department of Information Technology in Primary Schools in Cyprus at http://www.schools.ac.cy/klimakio/index.html (accessed on 31.04.2008)

The English National Curriculum at www.nc.uk.net (accessed on 25.05.2008)

The Nrich project at www.enrich.maths.org.uk (accessed on 29.05.2008)

The British Educational Communications and Technology Agency at www.BECTa.org.uk (accessed on 23.05.2008)

The Standards Site of the Department for Children, Schools and Families at www.standards.dfee.gov.uk (accessed on 24.05.2008)

The Department for Children, Schools and Families at www.dfes.gov.uk (accessed on 25.05.2008)

The Office of Her Majesty’s Chief Inspector of Schools at www.ofsted.gov.uk (accessed on 26.05.2008)

The Qualification and Curriculum Authority at www.qca.org.uk (accessed on 27.05.2008)