A Proposal for Developing
Online Collaborative Environment for Learning Mathematics

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Abstract: This paper addresses the issues for effective communication and collaboration in mathematics in online environments. The study documented in the paper explores students’ problem solving processes in computer environments by capturing students’ work done on the screen and analyzing these records on frame base. As a result of this close analysis, we documented evidence for students’ need for an online collaborative environment to study and learn mathematics. That documentation allows us to propose a need for new tools and to describe the features of these tools.

Keywords: Online learning environment, collaborative learning, developing learning environment, mathematics education.

Communication and collaboration are two of the essential processes in understanding and learning mathematics because they allow students to reflect, share and discuss their understandings of mathematical concepts and procedures (Ontario Ministry of Education, 2007). Gadanidis, Graham, McDougall, and Roulet, (2002) underline the importance of collaboration and communication by arguing “mathematical learning is a social activity that helps students learn from listening and sharing and also from watching the actions, movements, and manipulations of others” (p. 9). Paper-and-pencil method is the most preferred method for students to demonstrate their solution processes in face-to-face environments because students can easily share their solution procedures and describe what they did by drawing tables or sketching diagrams. As a consequence of this communication, they can improve their understandings and cognitive abilities. However, this approach is rather challenging in online environments, and the existence of “obstacles associated with text based communication interfaces, where it is difficult to expressed ideas with mathematical language and graphical representation” may prevent effective collaboration (Gadanidis et al., 2002, p. 15). This challenge in online collaborative learning environments and the possible solutions supported by technology for this challenge will be the main focus of our study.

Technology improves our ability in communication as seen in many online collaborative environments such as Knowledge Forum, Sakai, and Wikis; each provides different opportunities. For example, Knowledge Forum and Sakai provide effective tools (e.g. scaffolding, annotating, and the ability of following a thread) to structure online discussions whereas Wikis help to create a common wisdom by reshaping the others work and thoughts. Despite the sustained improvements in collaborative environments, we, as Mathematics Educators, need more tools for a better integration of technology in our curriculum. Gadanidis et al. (2002) reported this need as follows:

One of the key tools for an online learning environment for mathematics will be a shared whiteboard (and possibly synchronous voice and/or video). This is necessitated by the frequent need in mathematics for students and/or teacher to discuss ideas in free form while simultaneously editing mathematical

information. When you consider large number of exchanges back-and-forth in a typical mathematical “conversation” and the slowness of asynchronous replies, the need powerful synchronous tool comes to the fore (p. 9-10).

In addition to the need described above, we will discuss the opportunity of focusing on mathematical thinking process as well as its product in our proposed model by introducing a new method called “frame analysis method” after briefing literature related to the study.

**Computer Supported Collaborative Learning Environments**

Collaborative learning has situated its roots in the theory of distributed cognition introduced by Lev Vygotsky since 1930s as meta-analyzed by Morgan, Brickell, and Harper (2008). Distributed cognition describes how people interact with their environment including each other in order to advance their cognitive abilities, but not capacities. Sweller (1999) explained the limits of cognitive capacity of individual by introducing cognitive load theory and possible solutions for the effective use of this limited capacity. Collaborative learning enhanced by peer interaction not only in face-to-face settings but also in online environments provided evidence for a two-way benefit for learners (Tseng and Tsai, 2007). They analyzed online peer assessment and the role of peer feedback and demonstrated that students learn better by getting feedback as well as by providing feedback.

Technology can promote this collaboration by (1) providing cognitive tools to increase the ability of cognition and (2) providing advanced communication tools to increase the level of socialization. The former contains Dynamic Learning Systems, such as Geometer’s Sketchpad and Virtual Manipulatives, and Computer Algebra Systems, such as Maple and TI-Calculators whereas the latter includes Knowledge Forum, Sakai, and Wikis. As a result of these theoretical and practical attempts, new concepts and learning opportunities have emerged such as e-tutoring, online mentoring, e-coaching, e-moderating (Smet, Keer, & Valcke, 2007). Goos, Galbraith, Renshaw, and Geiger (2003) argue that technology can foster mathematical thinking processes such as conjecturing, justification, and generalization by enabling advanced computation and analysis of data and multiple representation forms. Moreover, they point at the re-organization effect of technology in addition to its amplification effects by exemplifying the increasing importance of spreadsheets and graphing tools against algebraic reasoning. How do mathematics educators evaluate these contemporary innovations? Do they really accept that these innovations reflect their needs and demands?

Mathematics educators have been interested in investigating the effectiveness of these cognitive tools, and many researches have been done in order to test their benefit (i.e. Forster, 2006; Galbraith, 2006; Kieran & Drijvers, 2006; and Threlfall et al, 2007). Regarding with the use of these tools, many scholars suggest that technology should develop students’ thinking capacity by using as a cognitive tool (i.e. Gadanidis et al., 2002; Goldenberg, 2000). In addition to thinking capacity or problems solving ability, focusing on thinking processes is another major interest emphasized by mathematics educators. This interest leads mathematics educators to develop a way to observe or at least to track students’ thinking processes.

Contemporary sophisticated communication tools enable us to observe and track students’ behavior and work while studying in computer environments. Hosein et al (2007) have employed the remote observation method to explore students’ understanding while studying in computer environments. Furthermore, eye-tracking and the index of cognitive activity have been suggested as more advanced versions of recording studies (Marshall, 1995, 2008). The former records learners’ eye movements while the student interacts with the computer and uses the data recorded as a result of more conscious attempts whereas the latter measures the change in pupil dilation and deals with the data based on reflex movements.

**The Study**

As many studies cited, students can effectively use online graphing tools, dynamic geometry and mathematics platforms, and virtual manipulatives while studying and learning mathematics. When we ask them to share their ideas and their work either with us or with their peers, they mostly use written documents created in the computer environment or scanned documents created by using paper-and-pencil method. In order to share the graphs and the solutions achieved by employing the features of the dynamic mathematics platform (Geogebra®, 2001) and virtual manipulatives (NLVM, 1999), they usually prefer the print-screen feature of computers and share these pictures with or without some editing on them. Despite our appreciation of their creativity and efforts during the study, a significant amount of messages they want to deliver is missing.

Given that we aim to contribute to the studies for increasing the effectiveness of communication and collaboration in online mathematics education, we designed a qualitative case study to collect data and employed the frame analysis method to analyze the data. In order to track students’ solution process in an online environment, we used Wink® software to capture students’ work done on the Geogebra dynamic platform. The screen casting software allows us to capture their work at any speed and never disturbs students while studying (see figure 1) as happens in video recording. We could analyze students’ work by comparing each frame of the record - what we mean by frame is the snapshots captured at any moment of the solution – and track every movement of mouse and entry of keyboard. This close analysis of students’ work enabled us to follow their solution processes and to investigate not only what they did but how they did it.

As illustrated in figure 1, one of the participants was asked to find the midpoint of a line segment joining two given points. The student’s drawing of the line segment (frame 99) and attempts to mark the midpoint can also be seen. Her first attempt was to draw another line segment equal to the half of the given line segment (frame 116) and to mark by using the end point of this segment whereas the second attempt was to draw a perpendicular line to the original line segment and passing through the end point of the half line segment which did not work. While looking for a better solution she realized that the end point of the half line segment changes and was shown on the algebra section of the software (on the left side) as she manipulated the line segment (frame 315). That is, when the line segments are coincided, the end point of the half line segment becomes the midpoint of the original line segment (frame 458). She took advantage of this to enter the coordinates of the midpoint by using the input line of the software and completed the solution.

Beyond tracking and monitoring students’ problem solving processes, we investigated how we could use this medium in communicating with the students and providing feedback to them. The screen-casting software used in this study, as in many others, allows us to zoom into any frame recorded and to annotate it. We used this feature of the software to deliver our messages and jotted our notes down on the desired frames. This feature made the communication easier because we could easily navigate the frames, describe the moment of action, and deliver the message. Besides, we could encourage the students to reflect on their work and think deeply – as demanded by mathematics educators – by asking questions based on their work. For example, one of the participants described the terms “inside the function” and “closer function” to describe the regions of the parabolas and to compare the functions. We engaged this student to find the proper term by asking clarifying questions (figure 2). At a later stage,

we decided to investigate how the students could use this feature to collaborate with each other. The participants were asked to analyze their peer’s works and try to understand what the others were doing. The result was encouraging because they could easily track their peer’s work and annotate as we expected.

![Annotating students' work](image1.png)  ![Annotating students' work](image2.png)

Figure 2: Annotating students' work

However, we must admit that these studies were performed under laboratory conditions, and further investigations are needed to generalize the results. Students may collaborate, discuss, and advance their knowledge by sharing these videos as well as by commenting and annotating on the others’ work. This communication and collaboration opportunity may allow students to learn by both from their peers and from evaluating their peers’ work (Tseng & Tsai, 2007) as well as gaining many interpersonal skills (Lee, 2006). The opportunity for collaboration, annotation, and commenting could become accessible to the public or to the authorized people depending on the need and purpose of the use. We are confident enough to declare that we have obtained some preliminary ideas for advancing our mathematical communication and collaboration in online environment, and we are ready to describe a tentative architecture for this environment, based on our needs.

### Challenges, Possible Solutions, and an Architecture

During the study, we experienced some problems as summarized below (see table 1), developed solutions for some of them, and completed the pilot testing. The results of the pilot testing quite encouraged us to dream of having a highly interactive online collaborative learning environment. Students who participated in the study did very well and started using software effectively without any hesitation. However, we strongly recommend that is should not be forgotten that these students are usually high achievers and good at both mathematics and technology.

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Table 1: Problems and possible solutions

Using multiple software such as Geogebra (cognitive tool), Wink (recording tool), and MSWord (editing tool) is the first challenge we have confronted although we could easily overcome this challenge because the students in this study were confident enough with the technology. In order to avoid this challenge in the future and to be able to access technologically challenged students, either full integration of these software or creation of an

interface to allow users easy access should be considered. Regarding with the other challenges, we suggest a specifically designed and sophisticated software, which allows to users to record and track at adjustable rates, with zoom in and zoom out features. Moreover, this software should allow multiple online users to access the system. Finally, despite the fact that we have tested this system in single-user take up online environment, an online secure environment which allows for multiple users is a definite suggestion.

Based on common experience and expectation and our experience from this study, we are at the stage of describing a tentative architecture of an online collaborative learning environment for mathematics education. Our proposed architecture contains two main modules: (1) administrative and (2) communication modules. The administrative module could be similar to the ones used in Sakai and Knowledge Forum - allow pre-registered users access to the server and perform the other security issues - whereas the communication module will contain some additional features such as more tools for communicating in mathematical language, recording features, and the existence of, or the opportunity for accessing online, cognitive tools.

As suggested by Gadanidis et al. (2002) and many other mathematics educators, in order to communicate in mathematical language; students, teachers, mathematics experts, and researchers need to access text, freehand drawing, and multiple representation tools (cognitive tools). Although all contemporary collaborating environments contain a text editor with a number of features, a text tool allowing symbolic representation is strongly suggested. For example, in one of the sessions we have documented, students were struggling with drawing number-line to find the intersection of sets. That is why, a freehand drawing tool, either as software or hardware (e.g. Smartboards), would be an asset. Furthermore, we suggest a cognitive tool such as Geometer’s Sketchpad, Geogebra, and the like or the possibility of accessing to the online versions of these kinds tools. Although this can also be achieved by creating a highly interactive and functional interface integrating all necessary software, but this type of integration may bring adaptation problems. Before moving on to describe the recording features, we should acknowledge that all these tools are already developed and the only thing left to do is to find a way to integrate them based on our needs.

Recording features are relatively new in educational settings although tested by many mathematics educators and employed by many educational researchers. We propose a video conferencing tool, which allows the users to communicate with each other, and a screen capturing tool with adjustable features. This screen capturing tool should allow the users to record their work at a specific rate as well as to track a recorded work at a desired rate so that the user can zoom in and reflect on a specific frame. Annotation of desired frames can help effective communication.

**Conclusion**

In order to contribute to the studies promoting online collaboration in mathematics education, we investigated and documented the needs for mathematics-specific tools - for more effective communication - and screen-casting tools - for tracking and monitoring students’ problem-solving processes. The use of screen-casting tools to track students’ problem-solving processes is relatively new in mathematics education and seems promising for the future. An effective integration of these tools into the current systems may promote collaboration and communication in mathematics in online environments.

**References**


